NEW ELEMENTS OF OPTICS.

PART IV, V, and VI.

BRITISH
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MUSEUM

NEW

ELEMENTS of OPTICS.

PART IV, V, and VI.

Containing the THEORY of

CATOPTRICS and DIOPTRICS
deduced from *Phyſical* and *Mathematical*
PRINCIPLES;

AND

It APPLICATION to the CONSTRUCTION and
USES of *Optical* INSTRUMENTS:

With all their IMPROVEMENTS to the preſent TIME;

The Whole illuſtrated with a great Variety
of COPPER-PLATE FIGURES.

By B. MARTIN.

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PREFACE.

AS my new and compendious Syſtem of Optics is now out of Print, and ſeveral new Improvements having been made of late in the Science, I thought it might be both agreeable and acceptable to the Public, to have the Whole drawn up in one ſmall ſynoptic View, with the philoſophical and geometrical Demonſtrations of every Part annexed. This I have done in the Compaſs of about 250 *Pages* 8*vo. The Whole is divided into* Six *Parts, each containing a peculiar Subject, or Branch of the Optic Science; my great View in this Publication, was to preſerve the Original and noble Diſcoveries of the great* HUGENIUS *in his Dioptrics, eſpecially thoſe relating to both Kinds of the Aberration of Rays, and the genuine Conſtruction of Optical Inſtruments; and to theſe I have added all that has been lately diſcovered in regard to the Correction of the Errors in the common Refraction of Rays, and the Conſtruction of the* Achromatic Teleſcope *conſequent thereupon; and have hinted many Improvements in the Structure of Optical Inſtruments,*

ments, rendering them as perfect as their reſpective Theories require. I hope the mathematical Reader will find no Cauſe of Complaint in regard to Perſpicuity, and a legitimate Proceſs, even in the abſtruſeſt Parts; and the philoſophical Reader, I preſume, will do me the Juſtice to allow that moſt of what he here finds tending towards a phyſical Explication of the Rationale *of any particular Subject or Propoſition is entirely new, and neceſſary to that End. The Whole is illuſtrated with all ſuch Experiments, and Copper-Plate Figures, as I thought requiſite to convey the beſt Ideas of theſe Things to a popular Reader. What Signatures of Humanity may be obſerved in the Execution, it is hoped will be compenſated by a benevolent Intention; for no one will be abſurd enough to ſuppoſe that any Author is actuated by* lucrative Motives *in ſuch Publications as theſe. To ſay all in one Word, I have here endeavoured to dreſs up a* SYSTEM *of* OPTICS, *theoretical and practical, for the Youth of the next Century in ſuch a Mode and Taſte, as I often wiſhed to have found for my own Uſe at the Beginning of this; and as ſuch I recommend it for their eaſieſt Introduction to Optical Science.*

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NEW ELEMENTS *of* OPTICS.

PART IV.

CHAPTER I.

The General THEORY of the ELEMENTS of DIOPTRICS, demonſtrated from Genuine PRINCIPLES.

387. THE firſt and fundamental Principle in OPTICS is, *that the Sines of Incidence are in a conſtant Ratio to the Sines of Refraction*, in all uniform Mediums, as *Air*, *Water*, *Glaſs*, &c. The Demonſtration of which is, therefore, in the firſt Place neceſſary.

Plate V. *Fig.* I.

388. Let DC be a Ray of Light incident from one Medium X upon the Surface HO of another Medium Y, (which ſuppoſe to be the denſeſt of the two,) and from the Point of Incidence C, let it be refracted to F, from its firſt Direction

DCM. This Refraction is owing to the attracting Power upon the Surface of the Medium Y, and acts upon the Ray in a perpendicular Direction, by which it acquires ſome new additional Force, and Velocity of Motion through the Medium Y.

389. Upon the Center C deſcribe the Circle AOPH, cutting the incident Ray in D, and drawing the Diameter ACP perpendicular to HO, let DL fall perpendicular thereto, and it will be the Sine of Incidence. Let DC or CE, repreſent the Space deſcribed in a given Time in the Medium X; and from F draw EF parallel to AB, to denote the new acquired Force in C. Now the Motions in the Directions CE and EF in the ſame Time, being compounded, will produce a Motion in a Direction found by joining CF; for then will CF be the Space deſcribed in the Medium Y, in the ſame Time DC (= CE) was paſſed over in the Medium X; and conſequently will be the *refracted Ray:* And therefore GI (perpendicular to AB) will be the *Sine of Refraction.*

390. Through F draw NM parallel to HO, and draw KE perpendicular to AB; then will BF = KE = DL = Sine of Incidence; and in the ſimilar Triangles CIG, CBF, we have CG : CF : : GI : BF. By which it appears, that the Sines of Incidence and Refraction BF or DL and GI are as the Velocities CF and CD (= CG) in the different Mediums inverſely, and *are therefore in a conſtant Ratio,* becauſe thoſe Velocities are ever invariably the ſame, as produced by the uniform Powers of Nature.

391. Therefore, on the other Hand, if the Ray FC be conſidered as paſſing out of a denſe

Medium Y, into a rarer Medium X, then in the Point C it will be deflected by the ſuperior Force of the Medium Y into the Direction CD, making DL : IG : : CF : CD, as before.

392. To apply this Doctrine to the Explanation of a New Theory of Optics, let AMD be the curved Surface of a refracting Medium Y, and B a radiant Point in a rarer Medium X, from which two Rays proceed, and fall upon the Curve in the Points M and m, indefinitely near to each other; theſe Rays will be ſo refracted as to croſs each other in a certain Point F, to determine which from the given Equation of the Curve, the Diſtance of the Radiant, and the Refractions of the Mediums is that Problem in Optics on which the whole Science, in all its Parts, depends.

Fig. II.

393. In order to a Solution thereof, let CM, Cm, be Rays of Curvature, or perpendicular to the Curve in the Points M and m. Upon MF and mF let fall the Perpendiculars CG, and Cg cutting FM in S. Alſo upon the incident Rays BM, Bm, continued out let fall the Perpendiculars CE, Ce; and on the Centers B and F deſcribe the ſmall Arches RM, MO; and put BM $= d$, ME $= a$, MG $= b$; the Arch MR $= s$, and the Arch MO $= t$; and laſtly, the Sine of Incidence CE to that of Refraction CG, as m to n, the Radius of Curvature being CM $= r$.

394. Then the Triangles MEC, MRm; MGC, MOm; MBR, QBe, are ſimilar, as is thus evident; if from the Right Angles RME, CMm, you ſubtract the Angle EMm, there remains the Angle RMm = EMC; and if from the Right Angles FMO, CMm, you take the Angle FMm, there will remain the Angle OMm = GMC.

$= GMC$. Theſe Triangles are therefore *equiangular*, and conſequently *ſimilar*.

395. Hence we are furniſhed with the following Analogies for determining the refracted Ray MF, *viz*. ME : MC : : MR : M*m*; that is, $a : r :: s : \frac{rs}{a} = Mm$.

396. Again, from the Triangles MGC and MO*m*, we have MG : MC : : MO : M*m*; that is, $b : r :: t : \frac{rt}{b} = \frac{rs}{a}$; therefore $sb = at$, and ſo $a : b :: s : t = \frac{bs}{a} = MO$.

397. In the Triangles BMR, BQ*e*, we have BM : BQ (= BE) : : MR : Q*e*; that is, $d : d + a :: s : \frac{ds + as}{d} = Qe$.

398. But C*e* : C*g* (: : CE : CG) : : *m* : *n* : : C*e* — CE : C*g* — CG : : Q*e* : S*g* : : $\frac{ds + as}{d} : \frac{nas + nds}{md} = Sg$.

399. Laſtly, the ſimilar Triangles EMO, FS*g*, give MO : S*g* : : MF : SF or GF; therefore MO — S*g* : MO : : MF — GF (or MG) : MF; that is in Symbols, $\frac{bmds - aans - ands}{amd} : \frac{bs}{a} :: b : \frac{mdbb}{mdb - aan - nda} = MF$, the focal Diſtance required.

400. As the Right Angles at E and G are both ſubtended by the ſame Hypothenuſe or right Line MC, 'tis evident that Line MC is the Diameter of a Semicircle MEGC paſſing through them, as in *Fig*. III. And if the Curve AMD be a *Circle*, then C will be the Center of it; and when the Point M is extremely near to the Vertex

A, we ſhall have ME = MG = MC, or $a = b = r$; in this Caſe the Theorem becomes $\frac{rmd}{md - nd - nr} = AF = f$, and the Point F or Focus of Refracted Rays is then in the Axis BC produced.

CHAP. II.

The General THEORY of the ELEMENTS of CATOPTRICS demonſtrated; with its APPLICATION to SPECULUMS.

401.

THE foregoing Theorem ($\frac{rmd}{md-nd-nr} = f$) is of a general Nature, and equally ſerves to CATOPTRICS as to DIOPTRICS, as will be evident if we only conſider, that a Ray of Light DC is in Part always reflected from every poliſhed Surface HO of *Water*, *Glaſs*, &c. in ſuch Manner that the Angle of Incidence DCA is ever equal to the Angle of Reflection ACQ in the ſame Medium X. For let the Velocity of the incident Ray DC be reſolved into two others LC and DL; the former of which being perpendicular to the reflecting Surface, is that only which is affected by the reflecting Force in or upon it; while the other Part DL being parallel to the ſaid Surface HO, is not altered, and will therefore cauſe that the Ray at all equidiſtant Points of Time, before and after the

Fig. I.

the Moment of Incidence at C, will be equidiſtant from the Perpendicular AC; and conſequently as the Times of deſcribing DC and CQ, muſt be equal, ſo alſo muſt the Lines DL and LQ, or *the Sine of Incidence* DL *is equal to the Sine of Reflection* LQ. All this follows from the *Laws of Colliſion* between elaſtic Bodies.

Fig. III.

402. Therefore if AMD be a reflecting Spherical Surface, and BM a Ray *diverging* from the Point P, and incident upon the *Speculum* in M; and if CM be continued to I, and the Angle IML made equal to the Angle IMB, then will LM be the *reflected Ray*, which produced, will cut the Axis in f, the *virtual Focus* of all diverging Rays from the Point B.

403. The above Theorem finds this focal Diſtance A f when properly qualified; thus, the *reflected Ray* ML is to be conſidered as a *negatively refracted one*, and conſequently the Members of the Equation where n occurs, muſt have their Signs changed; which will then appear in this Form $\frac{rmd}{md + nd + nr} = f$. But becauſe in Reflections, the Sine of Incidence m is equal to the Sine of Reflection (*or negative Refraction*) we have $n = m$, and ſo $\frac{rmd}{2md + mr} = \frac{rd}{2d + r} = f$. Which is the univerſal Theorem of Catoptrics.

404. If we ſuppoſe the Diſtance of the Radiant B to be infinitely great, then $\frac{rd}{2d} = \frac{r}{2} = f =$ AO, or the focal Diſtance will become AO $= \frac{1}{2}$ AC, the Radius, for parallel Rays HM.

405. If the Speculum AMD be *concave* inſtead of *convex*, the ſame Theorem ſerves with

the Sign of r (the Radius) changed, *viz.* $\frac{-rd}{2d-r}=f$, which ſhews that in ſuch a concave Speculum, when d is infinite, $\frac{-r}{2}=f=\text{AO}$, as before; but now a *real Focus*, and where the Sun Beams are actually collected, and burn.

406. It appears alſo that the Focus of a *concave* Mirrour will always be real (or before it) while $2d$ exceeds r; but when $2d=r$, then the focal Diſtance is infinite; and when $2d$ is leſs than r, the ſaid Focus is again virtual or imaginary, and behind the Speculum.

407. If the Speculum be a *plain Surface*, or AMB a right Line, then r is infinite, and $d=f$; as in a *Looking-Glaſs*, we ſee the Object and its Image at equal Diſtances from it.

408. By ſuppoſing the Curve AMD to be one of the *Conic* SECTIONS, *viz.* an ELLIPSIS, PARABOLA, or HYPERBOLA, ſeveral beautiful Theorems might be from thence derived; and alſo the Doctrine of CAUSTICS in a general Manner by Reflection and Refraction have been explained; but as theſe are Speculations of little Uſe in Practice, and having treated the whole Subject at large in Vol II. of the INSTITUTES *of* ARTS *and* SCIENCES, I ſhall not inſiſt upon it here.

409. One Thing, however, is too curious not to be taken Notice of, and that is, that the Theorem $\frac{rd}{2d+r}=f$, contains a Demonſtration, that the Axis of the Speculum AMD, is *harmonically* divided in the Points B, A, f, and C. *Fig.* III. For by the Theorem we have $rd=2df+rf$; thence $rd-df=df+rf$, and therefore $d+r:d::$

$r : d :: r - f : f$, or BC : AB : : fC : fA, which is well known to be a *muſical Diviſion* of the Line BC. If the Speculum be concave, r will be a negative, and the Analogy is $d - r : d :: r + f : f$, or AB : CB : : fA : fC.

Plate VI. *Fig.* I. & II. 410. Let CB be the Axis of the Speculum AM, and C its Center; and let OB be an Object at a given Diſtance; from the extreme Point O, draw OC, interſecting the Speculum in G; this will be the Axis to all the Rays flowing from the Point O; of theſe OA is that Incident upon the Vertex A, which will be reflected into AD, making the Angle OAB = DAB. This reflected Ray AD (continued out) will interſect the Axis COG in the Point E, which will therefore be the Focus of all the Rays from O, and conſequently the Image of the Point O will be formed in E; and it has been ſhewn that f is the Focus of Rays from B, and therefore the Image of the Point B will be formed in f; therefore E f will be the Image of the Object OB.

411. Hence it is evident by Inſpection, *That the Object* OB *and its Image* E f *are ſeen under the ſame or equal Angles from the Center* C *of the Speculum.* For OCB = ECf.

412. It is further evident, *That the Object and Image are ſeen under equal Angles from the Vertex* A *of the Speculum.* For OAB = DAB = EA f.

413. Hence likewiſe it follows, that the right-angled Triangles OAB and EA f are ſimilar, and therefore it is OB : E f : : AB : A f; or *the Object and its Image have the ſame Proportion as their Diſtances from the Vertex* A *of the Speculum.*

414. Let this Proportion of the Object to the Image be as o to 1; then we have o : 1 : : AB : A f : :

$Af : : d : f = \frac{dr}{2d+r}$, which will give $d = r \times \frac{O-I}{2I}$; which plainly ſhews, *that no Object can be magnified by a convex Speculum*; for in that Caſe I would be greater than O, and ſo d would be a negative Quantity, which is impoſſible. *Therefore ſuch a Mirrour always diminiſhes Objects.*

415. But in a *concave Speculum*, it is $d = r \times \frac{O+I}{2I}$; hence by putting I = O, we have $d = r$, which ſhews, *that the Image is equal to the Object, when placed in the Center* C *of the Speculum*; *and then they are both together in that Point.*

416. Again, in the *concave Speculum*, while O is greater than I, 'tis evident d will be greater than r; that is, *while the Object is placed at a greater Diſtance from the Mirrour than the Centre* C, *the Image will be leſs than the Object.*

417. But if O be leſs that I, d will be leſs than r; that is, in other Words, *the Object will always be magnified when at a leſs diſtance from the Speculum than the Center.*

418. Becauſe $d : \frac{r}{2} : : O + I : I$, all the while d is leſs than r, but greater than $\frac{1}{2} r$, it is plain I may be in any Proportion greater than O, *or the Object may be magnified in any given Degree, in a poſitive Image, or before the Mirror.*

419. But when $d = \frac{1}{2} r$, or the Object is placed in the ſolar Focus, then O + I = I, or I is infinite in Compariſon of O; that is, *the Object is then infinitely magnified at an infinite Diſtance.*

420. Laſtly, when d is leſs than $\frac{1}{2} r$, then O + I is leſs than I; conſequently O muſt be negative,

gative, and it will be $d : \frac{1}{2} r :: I - O : I$; and therefore the Object will be magnified till $d = O$; when $I = O$, *or the Object and Image are equal upon the Surface of the Mirrour.*

421. From the Conſtruction of the Figures, it appears, that the Image E f in the *convex Speculum* will always be *erect*; and in the *Concave*, the Image will be *erect* when ſeen behind the Glaſs; but while it is before the Glaſs, it will be always *inverted*, becauſe the Rays OE and B f croſs each other (in the Center C) before the Image is formed; whereas in the *Convex*, they do not interſect at all.

CHAP. III.

An INVESTIGATION of an *Univerſal* DIOPTRIC THEOREM, with its APPLICATION to all Kinds of LENSES.

422.

WE next proceed to adapt the ſame Theorem to a Lens, and apply it to all the Branches of DIOPTRICS. As a Lens has *two Surfaces*, and we have conſidered the Refraction only at *one*, out of a rare into a denſer Medium; we muſt now, on the other Hand, ſee how a Ray MN, which in a denſe Medium Y proceeds towards a Point F in a rarer Medium X, but meeting with the ſpherical Surface

Plate I. *Fig.* IV.

face ND, is refracted into the Direction N f, to interſect the Axis in the Point or Focus f.

423. When theſe two refracting Surfaces AM, ND, are near to each other, they form a Lens AMND, whoſe Axis is BADF, and to find the Point f, or focal Diſtance D f of the Ray BM after being twice refracted at M and N, is the General Problem of DIOPTRICS.

Fig. V.

424. To the Solution thereof, we obſerve the above Theorem (400) gives $mdf - mdr = nrf + ndf$ (making $MF = f$) from whence we deduce this other Equation, $\frac{m}{n} = \frac{r+d}{f-r} \times \frac{f}{d} = \frac{BC}{CF} \times \frac{AF}{AB}$, which gives this univerſal Canon in Words, *viz.* *The Ratio of the Sine of Incidence to the Sine of Refraction is compounded of the Ratio of the Diſtances of the conjugate Foci* B *and* F *from the Center* C, *and of the Ratio of their Diſtances from the Vertex* A.

425. This Rule being general, finds the Focus f after the ſecond Refraction at N; for let D f = f, the Radius GD = r, and the Thickneſs of the Lens AD = t; then we have for the Refraction out of a denſe Medium into a rarer $\frac{n}{m} = \frac{FG}{\mathrm{f}G} \times \frac{\mathrm{f}D}{FD} = \frac{f+\mathrm{r}-t}{t+\mathrm{r}} \times \frac{\mathrm{f}}{f-t}$; from whence we get $\mathrm{f} = \frac{nf\mathrm{r} - nt\mathrm{r}}{mf - mt + m\mathrm{r} - nf + nt}$ = D f the focal Diſtance required.

426. If the Thickneſs t of the Lens be inconſiderable, we have $\frac{nf\mathrm{r}}{mf + m\mathrm{r} - nf} = \mathrm{f}$; and this will give $f = \frac{m\mathrm{f}\mathrm{r}}{n\mathrm{r} + n\mathrm{f} - m\mathrm{f}} = \frac{mdr}{md - nd - nr}$

(400) which Equation reduced, gives

$$\frac{nd r\mathrm{r}}{mrd - nrd + md\mathrm{r} - nd\mathrm{r} - nr\mathrm{r}} = \mathrm{f}.$$

427. Let $\frac{m-n}{n} = a$, then this Equation becomes $\frac{dr\mathrm{r}}{ard + a\mathrm{r}d - r\mathrm{r}} = \mathrm{f}$, the *Univerſal Dioptric* THEOREM. Q. E. I.

428. By theſe Theorems the Focus F by one Refraction may be found in a Medium of any given Denſity; and by two Refractions in a Lens of any given Figure. To illuſtrate this by Examples:

429. The focal Diſtance AF $= f$, of Rays HM parallel and near to the Axis in a Body of GLASS, is found by the Theorem (401), where putting $d = \infty$, and $m : n :: 3 : 2$, we have $\frac{mr}{m-n} = 3r$ $=$ AF.

430. The Focus f of parallel Rays falling upon a GLOBE of *Glaſs* is found by Theorem (425); for $f = 3r$, $t = 2r$, and $\mathrm{r} = r$, therefore $\mathrm{Df} = \mathrm{f}$ $= \frac{1}{2}r$.

431. For a HEMISPHERE of Glaſs, we have $t = r$, and $\mathrm{r} = \infty$, therefore $\mathrm{Df} = \frac{4}{3}\mathrm{r}$. If the plain Side of the Hemiſphere be expoſed to the Rays, then r, f, and d being infinite, we have $\mathrm{Df} = 2\mathrm{r}$.

432. The plain Side of a *Plano-Convex* LENS turned to parallel Rays, has $\mathrm{Df} = 2r$, alſo. But the convex Side gives $f = 3r$, and $\mathrm{r} = \infty$, and ſo $\mathrm{Df} = 2r - \frac{2}{3}t$.

433. If the LENS be a *double and unequally convex One*, the Theorem for parallel Rays gives $\frac{r\mathrm{r}}{ar + a\mathrm{r}} = \mathrm{f}$, becauſe $d = \infty$.

434. If the Lens be *equally convex*, the Theorem is $\frac{dr}{2ad - r} = \mathrm{f}$; for in this Caſe $r = \mathrm{r}$.

435. If r be infinite, the Lens is a *Plano-Convex*, and its focal Diſtance is $\frac{dr}{ad - r} = \mathrm{f}$. In Caſe of parallel Rays, where d is infinite, the focal Diſtance of the double and equally convex Lens is $\mathrm{f} = \frac{r}{2a}$, and in the Plano-Convex $\mathrm{f} = \frac{r}{a}$.

436. If one of the Radii be negative (as r,) the Lens is a *Meniſcus*, and the Theorem becomes $\frac{dr\mathrm{r}}{ad\mathrm{r} - adr - r\mathrm{r}} = \mathrm{f}$; and for parallel Rays $\frac{r\mathrm{r}}{a \times \overline{\mathrm{r} - r}} = \mathrm{f}$.

437. If each Radius be ſuppoſed negative, the Lens is an *unequally concave One*, and the Theorem is $\frac{dr\mathrm{r}}{-adr - ad\mathrm{r} - r\mathrm{r}} = \mathrm{f}$, always negative. And for parallel Rays, $\frac{r\mathrm{r}}{-a \times \overline{\mathrm{r} + r}} = \mathrm{f}$, or $\frac{r\mathrm{r}}{a \times \overline{\mathrm{r} + r}} = -\mathrm{f}$.

438. When the Radii are negative and equal, the Lens is *equally concave*; and the focal Diſtance is $\frac{dr}{-2ad - r} = \mathrm{f}$; and for parallel Rays $\frac{r}{-2a} = \mathrm{f}$, or $\frac{r}{2a} = -\mathrm{f}$.

439. If one Radius (r) be infinite, and the other negative, the Lens is *Plano-Concave*, and its focal Diſtance is $\frac{dr}{-ad - r} = \mathrm{f}$; or $\frac{dr}{ad + r} = -\mathrm{f}$,

— f, and that of parallel Rays is $\frac{-r}{a} = \mathrm{f}$, or $\frac{r}{a} = -\mathrm{f}$.

440. If the Radius (*r*) of the firſt Surface, or that towards the Radiant, be negative and leſs than the Radius (r) of the ſecond Surface, the Lens will be a *Concavo Convex*, and the Theorem for its focal Diſtance becomes $\frac{d r \mathrm{r}}{a d r - a d \mathrm{r} - r \mathrm{r}} = \mathrm{f}$, always negative, and for parallel Rays it will be $\frac{r \mathrm{r}}{a \times \overline{r - \mathrm{r}}} = \mathrm{f}$.

441. In any of the above Caſes, if the focal Diſtance for *converging Rays* be wanting for any particular Lens, it is only making the Diſtance of the Radiant (*d*) negative, or altering the Sign of the Term where it is found, and it will be given by the Theorem (433) for that Lens. Thus the Theorem $\frac{dr}{2ad - r} = \mathrm{f}$ becomes $\frac{-dr}{-2ad - r} = \mathrm{f}$, or $\frac{dr}{2ad + r} = \mathrm{f}$, for converging Rays.

442. In a Lens, as we have formerly obſerved (29), an Object and its Image are ſeen under the ſame Angle from the Vertex; therefore if O and I repreſent the Ratio of the Object to the Image, in regard to their linear Dimenſions, then O : I : : $d : f = \frac{dr}{2ad - r} = \frac{dr}{d - r}$ (becauſe $2a = 1$, nearly;) therefore we have $d = \frac{r\mathrm{O} + r\mathrm{I}}{\mathrm{I}}$; conſequently when O = I, then $d = 2r$, which is of great Uſe to know, as by the *Camera Obſcura* it may be required to draw Objects juſt as big as the Life.

443. When *d* is infinite, the Theorem becomes $d = \frac{rO}{I}$; which ſhews the Image I is then nothing in Compariſon of the Object O; as is the Caſe of the *Solar Image* compared with the *Sun* itſelf.

444. If the Image I be negative, or on the ſame Side of the Lens with the Object, then the Theorem is $d = \frac{rI - rO}{I}$, which is the Caſe of a *Reading Glaſs*, wherein the Image *is always larger than the Object*, as its apparent Place is farther from the Glaſs.

CHAP. IV.

The THEORY of REFRACTIONS by the PRISM both of *Homogeneal* and *Heterogeneal* LIGHT.

445.

NOTWITHSTANDING we have treated pretty largely on the Nature of thoſe two Sorts of Aberrations which ſo much diſturb Viſion by refracted Light, yet there is required ſome further Illuſtration of this Doctrine, that it may be more clearly conceived how they exiſt in the real Conſtruction of *Microſcopes*, *Teleſcopes*, and other Optical Inſtruments, and the Reaſon of preventing, correcting, or removing them will, of Courſe, more fully appear. At the ſame Time ſuch a Diſſertation will ſerve for a proper *Tranſition* from the *Vulgar* to the *New* ELEMENTS of OPTICS.

446. To this End we muſt conduct our Speculations by the Theory and Experiments of the PRISM, which therefore we ſhall here premiſe. *Fig.* VI. Let GIH be the Section of a Priſm whoſe refracting Angle at I is but ſmall, *viz.* leſs than 30 Degrees. And let QA be a Ray of *homogeneal* Light incident upon the Priſm at A, and from thence refracted through its ſubſtance to the other Side at C, where it is again refracted into the Air in the Direction CS. Produce the incident Ray QA to R, and the refracted Ray SC to T, interſecting the other in the Point F: laſtly, let BK be perpendicular to the Side GI in A, and DL perpendicular to the other Side IH in C, theſe two Perpendiculars will interſect each other in E.

447. Since the refracting Angle I of the Priſm is ſmall, the Angles of Incidence and Refraction BAF, BAC, will be proportional to their Sines, that is, $BAF : BAC :: m : n$; and therefore diſjointly, we have $BAF - BAC\ (= CAF) : BAC :: m - n : n$. After the ſame Manner at the Emergence on the other Side, it is $TCA : DCA :: m - n : n$. Wherefore $CAF + TCA : BAC + DCA :: m - n : n$.

448. Now $BAC + DCA = AED$ which is equal to the refracting Angle I of the Priſm, as is thus evident; let the Perpendicular IE interſect the Side GI in D, then in the right angled Triangles ICD, and DEA, the two Angles $I + D = 2$ right Angles $= AED + D$; there-

fore AED = I. Alſo the Angles CAF + TCA = RFC or TFA = the Total Refraction of the Ray. Therefore AED : RFC : : *n* : *m* — *n*; Or *the Refraction Angle* I *is to the total Refraction* RFC *as the Sine n is to the Difference of the Sines m — n. Therefore theſe Quantities are in a conſtant Ratio.*

449. And becauſe the Angle I is conſtant, *therefore the total Refraction of a Ray in every Incidence, will be the ſame, or invariable.*

450. Conſequently when a Ray enters the Priſm in a Direction perpendicular to one of its Sides, and ſo paſſes to the other without Refraction, it will there be refracted into the Air in the Angle RFC or RCS ſtill the ſame as before. In this Caſe the Points E and F coincide with the Point C. *Fig.* VII.

451. Therefore in Lenſes equally or unequally convex or concave, while their *focal Diſtances and Apertures are the ſame*, the total Refraction will be the ſame in them all; and, of Courſe, the *Diameter of the Circle of Aberrations* in the Focus of a Lens can vary only with the Diſtance of that Focus ariſing from the different Inclination of the Rays to the Axis of the Lens. Since it is known, the Angle of Diſſipation is, in the ſame Priſm or Lens, conſtantly the ſame.

452. Hence alſo it will follow, that any two homogeneal Rays, after Refraction from the Priſm, will have the ſame Inclination to each other as they had before, or at their Incidence upon it. Thus let QA *q*A, be the two incident *Fig.* VIII. Rays inclined to each other in an Angle Q A *q*; in the Priſm they are refracted to C and *c*; and from thence into the Air in the Directions CS, *cs*; which refracted Rays being produced back-

ward to interſect the reſpective incident Rays in F and f, will make the Angles AFC, A f *c*, equal, becauſe RFS and *r* f *s* will always be ſo (448). Alſo the Ray *q*A, continued out, interſects the refracted Part SC of the other Ray in M, and the two refracted Rays S C, *s c*, produced, interſect in L; by which Means there are formed two Triangles AFM, and MfL, in which the Angles at F and f, and alſo at M being equal, the remaining Angle FAM, and fLM will be alſo equal; but theſe Angles meaſure the Inclinations of the Incident and refracted Rays, therefore the Propoſition is evident.

Fig. IX. 453. Hence if AB be a Lens, and A Q, A *q*, two Rays of *red Light*, one of which is parallel to the Axis, the other inclined thereto, and interſecting it in the Point *q*, theſe Rays after Refraction through the Lens will have the ſame Inclination to each other, and interſect the Axis in their reſpective *Foci* F and f; that is, the Angle FAf will always be equal to the Angle QA*q*.

454. For the ſame Reaſon, if we ſuppoſe QA, *q*A to be Rays of *violet Light*, as they are moſt refrangible, they will be refracted to the Axis ſooner in their *Foci* S, *s*, making the Angle SA*s* = QA*q*, as before. Therefore ſince the Angle FAf = SA*s*, if we take away the Angle FA*s*, common to both there will remain the Angle fA*s* = FAS, or *the Angle of Aberration in Rays parallel or oblique to the Axis is always the ſame in the ſame Lens*, and with a given *Aperture and focal Diſtance*.

455. On the Point S erect the Perpendicular SD, and on *s* the Perpendicular *sd*. Then will SD be the *Lateral Aberration* of parallel Rays QA; and *sd* the ſame Aberration for Rays di-

verging from any Point q in the Axis. Then it is evident AS : AG : : ds : GH; and becauſe the Angle fAs = FAS, it is GH = SD, nearly; alſo becauſe when the Angle As B is ſmall, we have As and AG nearly equal to Bs, and BS; therefore SD : sd : : BS : Bs, that is, *theſe Lateral Aberrations will be as their Diſtances from the Lens* AB.

456. But the *Longitudinal Aberrations* Fs, fs, *will be as the Squares of the Diſtances*, BS, *and* Bs, *from the Lens.* For put the Radiant Diſtance Bq = d; focal Diſtance of parallel Rays BF = r; conjugate focal Diſtance Bf = f; ſemiaperture of the Lens AB = a; and FS = n. Then BS = $r - n$, and by common Optics we have $Bf = f = \frac{dr}{d-r}$; and ſince the Angle GAS = QAq = AqB, we have Bq : AB : : AG (or BS) : SG; or $d : a :: r - n : \frac{ar - an}{d} =$ SG. And becauſe AB : GS : : sB : sS we have AB — GS : AB : : sB — sS : sB; that is, $\frac{ad - ar + an}{d} : a :: r - n : \frac{dr - dn}{d - r + n} = Bs$; this ſubducted from B$f$, leaves $\frac{ddn}{dd - 2dr + rr + dn - rn} = sf$, which is to FS = n, as dd is to $d^2 - 2dr + rr + dn - rn$.

457. But becauſe the Quantity n is very ſmall, the Terms wherein it is found may be neglected, and the above Ratio becomes sf : FS : : dd : $dd - 2dr + rr$; therefore $\sqrt{sf} : \sqrt{FS} :: d : d - r :: f : r :: Bf : BF$; or $sf : FS :: Bf^2 : BF^2$. Q. E. D.

458. The Aberrations which have been hitherto conſidered and treated of, are of the *colorific Kind*; but the ſame Demonſtrations hold good for thoſe ariſing from the *ſpherical Figure*, by ſuppoſing F and *f* the focal Points of the Rays AQ and A*q*, when the Point A is indefinitely near the Axis or Vertex B of the Lens, and S and *s* the *Foci* of the ſame Rays at the Diſtance AB. For ſince in this Caſe the Angles $SAs = QAq = AqB$, all Things will proceed as before.

459. Alſo in this Caſe the Angles of Aberration SAF and *s*A*f* are equal, for by Compoſition of Ratios, we have $\frac{fs}{FS} = \frac{fs}{sd} \times \frac{sd}{SD} \times \frac{SD}{FS} = \frac{Bf}{AB} \times \frac{sd}{SD} \times \frac{AB^{*}}{BF} = \frac{Bf}{BF} \times \frac{sd}{SD} = \frac{Bf^2}{FB^2}$, (457.) therefore $\frac{sd}{SD} = \frac{Bf}{FB} = \frac{sB}{SB} = \frac{sA}{SA}$, becauſe of the Smallneſs of the Parts *fs* and FS, and of the Angle A*s*B; hence $\frac{sd}{As} = \frac{SD}{AS}$, or the Angles $fAs =$ FAS, becauſe any Angle FAS is directly as its Subtenſe SD, and inverſely as the Radius SA.

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* **By Similarity of Triangles AFB, DFS, and A*f*B, *dfs*.**

CHAP. V.

Sir ISSAC NEWTON's Method of inveſtigating the QUANTITY of each Kind of ABERRATION in a given *Plano-Convex* LENS.

460.

HAVING demonſtrated the Proportion of both theſe Sorts of Aberrations, we next proceed to ſhew what their real Quantity is in a given Plano Convex Lens, as ABD, whoſe Center is C, Radius AC, Semiaperture AE, and Thickneſs BE; and let QA, DI, be Rays parallel to the Axis CF incident upon the Lens at A and D equidiſtant from the Axis; theſe Rays will be refracted ſo that the *Red Light* will go to F, and the *Violet* to G; produce AG and DG to interſect AF and OF in P and R; and draw the Line PR, which produce to L, and it will be perpendicular to the incident Ray QA (continued out) at H; then producing AC to L, we have HO ($=$ AE) the Sine of the Incident Ray, and LP, LR the Sines of the leaſt and moſt refracted Rays AF and AR.

Plate V. *Fig.* X.

461. Let $HO = I$, $LP = L$, and $LR = G$, then $HP = L - I$, $HR = G - I$, very nearly; alſo $PR = G - L$; therefore it is $PR : HP :: G - L : L - I$, and by doubling the Conſequents, we have $PR : 2HP :: G - L : 2L - 2I$; but $2HP = AD - PR$; therefore conjointly,

jointly, PR : AD : : G — L : G + L — 2 I. By Sir ISAAC NEWTON's Experiments, the Sines I, L, G, are as the Numbers 50, 77, 78. Therefore PR : AD : : 1 : 55, or PR $= \frac{AD}{55}$.

462. Therefore alſo the Angle of Aberration PAR is $\frac{1}{55}$ of 2 HAR or 2 HAO = 2 AOE. Conſequently when AE or AB is given, the *Circle of Aberration or its Diameter* PR is given likewiſe in a given focal Diſtance BO of the Lens.

463. Again, becauſe CF : AF (or FB) : : 77 : 50; and CG : AG (or BG) : : 78 : 50; we have BF : BG : : 28 : 27, and therefore FG $= \frac{BF}{28}$. And theſe Dimenſions of this Sort of Aberration are the ſame in Lenſes of any Shape, if they have the ſame Aperture and focal Diſtance (454). And being determined for Rays parallel to the Axis, they are known for any other that are inclined thereto, viz. *diverging* or *converging Ones.*

464. As to the *Calculus* for the Aberration reſulting from the Figure of the Glaſs, Sir ISAAC NEWTON has given us the following. Let NBMG be the Plano-Convex Lens, AN, HM two Rays parallel to the Axis incident on the plain Side, and refracted into the Axis at the Point K, the Focus of the Lens being F, and the Aberration ſought KF. Let C be the Center of the Lens, and CE perpendicular to KN produced. Then the Sine of Incidence is to the Sine of Refraction as NG to CE, or as n to m, out of Glaſs into Air. *Fig.* XI.

465. Now putting CB $= r$, GB $= x$, and CK $= z$, we have, from the Nature of the Circle, $NG^2 = 2rx - xx$, to which add $GK^2 = (\overline{z-r-x}^2 =) z^2 + 2xz - 2rz + x^2 - 2rx + r^2$,

$+ r^2$, and the Sum will be $NK^2 = z^2 + 2zx - 2rz + rr$. And becauſe of the ſimilar Triangles CEK, NGK, it is NG : CE : : NK : CK : : $n : m$; therefore $n^2 : m^2 : : (NK^2 : CK^2 : :)\ z^2 + 2zx - 2rz + r^2 : z^2$. This Analogy (putting $\frac{m^2 \times \overline{r - x}}{m^2 - n^2} = s$) gives $z^2 = 2zs - \frac{m^2 r^2}{m^2 - n^2}$ or $z^2 - 2zs = - \frac{m^2 r^2}{m^2 - n^2}$; and compleating the Square, and extracting the Root we have $z = s + \sqrt{ss - \frac{m^2 r^2}{m^2 - n^2}}$.

466. By reſtoring the Value of s, and reducing the radical Part to an *infinite Series*, we ſhall have $z = \frac{mr}{m - n} - \frac{m^2 x}{mn - n^2} - \frac{m^3 x^2}{2n^3 r} - \frac{m^5 x^3}{2n^5 r^2}$, *&c.* and becauſe when $x = 0$, we have $z = \frac{mr}{m - n} =$ CF, therefore CF — CK $= \frac{m^2 x}{mn - n^2} + \frac{m^3 x^2}{2n^3 r} + \frac{m^5 x^3}{2n^5 r^2}$, *&c.* = KF, the Aberration ſought.

467. Hence when x is very ſmall, thoſe Members of the Series where its Powers x^2, x^3, are found, will be inconſiderable, and the Error will be ſimply KF $= \frac{m^2 x}{mn - n^2}$; and if $m : n : : 3 : 2$, we have KF $= \frac{9}{2}$ BG, as we have ſaid (82.)

468. If we put $NG = y$, then becauſe $NG = BG \times \overline{BC + CG} = BG \times 2BC$ nearly, it is $y^2 = 2rx$, or $\frac{y^2}{2r} = x$, and therefore $\frac{m^2}{mn - n^2} \times \frac{y^2}{2r} =$ KF; and conſequently the Error KF is as the Square of the Semiaperture NG of the Lens,

469. If the Ray ANK be given in Poſition, and *a n* be any other parallel Ray nearer to the Axis, on the other Side of which, let n *k* be the refracted Part cutting the Axis in *k*, and the refracted Ray NK in Q; and from Q draw Q*o*, perpendicular to the Axis, then will the Line K*o* become a *Maximum*, when the Ray *a n* is half the Diſtance of AN from the Axis.

470. For draw *ng* perpendicular to the Axis; and put $ng = v$, $Ko = s$, $GK = f$, and $KF = b$; and then $NG^2 : ng^2 :: KF : kF$ (468.) or $y^2 : v^2 :: b : \frac{bvv}{yy} = kF$; therefore $KF - kF = kK = b - \frac{bvv}{yy} = \frac{byy - bvv}{yy}$.

471. Again; $GK : GN :: Ko : Qo$, and therefore $Qo = \frac{ys}{f}$, alſo $gn : GK$ ($= gk$ nearly) $:: Qo : ok = \frac{ys}{v}$, to which add K*o*, and we have again $Kk = \frac{vs + ys}{v}$; whence $\frac{vs + ys}{v} = \frac{byy - bvv}{yy}$, and then dividing by $v + y$, and reducing the Equation, we have $s = \frac{bvy - bvv}{yy}$.

472. By making the Fluxion of this Equation $= o$, or $s = \frac{b\dot{v}y - 2bv\dot{v}}{yy} = o$, we have $b\dot{v}y - 2bv\dot{v} = o$; and thence $by = 2bv$, or $y = 2v$, that is, $NG = 2ng$.

473. This Line K *o*, when greateſt, is about $\frac{1}{4}$ of the Error KF, for if in the Equation for *s* above, you write $2v$ for *y*, there will ariſe $\frac{1}{4}b = s$.

474. Alſo becauſe CF — CB = BF = GK, nearly; therefore GK $= \frac{nr}{m-n}$; whence GK $\left(= \frac{nr}{m-n}\right)$: GN (= y) : : K o $\left(= \frac{1}{4}\ \text{KF} = \frac{m^2y^2}{8mnr - 8n^2r}\right)$: Qo $= \frac{m^2y^3}{8n^2r^2}$.

475. If the Arch BM = BN, and Bm = Bn, then the Rays incident on M and m being refracted, croſs each other in P, making the Space PQ = 2 Qo $= \frac{m^2y^3}{4n^2r^2}$. And moreover it appears that all the Rays between N and M are ſo refracted as to paſs through the Space PQ, and alſo that the Circle upon the Diameter PQ, is the leaſt Space poſſible in which all the Rays can be congregated; and therefore this Space is to be eſteemed the true Focus of the Lens or Place where the Image of the Object is formed by a Concourſe of all the Rays which fall between N and M, parallel to the Axis of the Lens. For it is evident, all the Rays between m and n paſs through PQ; alſo all the Rays between N and m, and between M and n, muſt paſs through PQ, therefore none can paſs on the Out-ſide of the Line PQ; nor can they be contained in a leſs Space, becauſe the Rays NK, MK interſect the external Rays mk, nk, in the Points P and Q, by which that Space PQ is limited.

476. If the Aperture NMB of the Lens, be encreaſed or diminiſhed, the lateral Error PQ will be as y^3, or as the Cube of the Breadth of the Aperture NM. But if the Aperture remains the ſame, the ſaid Error will be reciprocally as r^2, or as CB^2, and therefore as BF^2, ſince CB and BF are in a given Ratio. But when neither the

Radius nor Aperture of the Lens is conſtant, the Error will be as $\frac{y^3}{r^2}$, or as $\frac{NM^3}{BF^2}$; as appears from its Value $\frac{m^2 y^3}{4n^2 r^2}$, wherein the Part $\frac{m^2}{4n^2}$ is conſtant.

477. In different Sorts of Glaſs, or ſuch as have the Ratio of m to n variable, if the Aperture continue the ſame, the Error PQ will be $\frac{\mathrm{m}^2}{4n^2r^2}$. Or if $n = 1$, in every Sort of Glaſs, then this Error in Lenſes of the ſame Aperture will be as $\frac{m^2}{4r^2}$ to $\frac{\mathrm{m}^2}{4\mathrm{R}^2}$, or as $m^2\,\mathrm{R}^2$ to $\mathrm{m}^2\,r^2$. And in Lenſes of equal Aperture and focal Diſtance, the Errors will be as $m^2 : \mathrm{m}^2$.

478. Before I conclude this Subject it may
Fig. XII. be proper to obſerve, that a Ray of Light EF incident obliquely on the Surface AB of any refracting Medium ABCD, (bounded by parallel Surfaces) is diſſipated into the Beam GFH, which is by the ſecond Surface DC refracted again into a coloured Beam of parallel Rays IGHK, and in a Direction parallel to that of the incident Ray EF. For the *red Ray* FG will be equally refracted at F and G; therefore GI will be parallel to FE, and for the ſame Reaſon, the *violet Ray* HK is parallel to FE, alſo. Therefore any Object at E will (when the refracting Medium AC is conſiderably thick) appear ſomewhat coloured; and its apparent Place will be at L in the Direction KL parallel to EF.

NEW ELEMENTS *of* OPTICS.

PART V.

CHAPTER I.

The DESCRIPTION and STRUCTURE of the EYE, and the THEORY of *Natural* VISION by CALCULATION.

479.

THE general Manner in which Viſion is performed in the Eye has been already explained in Part I, (Art. 34, *&c.)* But we ſhall now give a more exact and particular Deſcription of the EYE and of its ſeveral Parts, with a Calculation of the various Refractions of the Rays of Light through the ſeveral Humours, for forming the Images of Objects upon the *Retina* at the Bottom of the Eye.

Plate VI. *Fig.* III. 480. To this End I have here repreſented a Section of the *Human Eye* in its true and natural Magnitude, which conſiſts of two Segments of two different Spheres, *viz.* one larger, as BNB, and a leſſer BIB. The larger Segment conſiſts of three Tunics or Coats, of which the outmoſt is of a hard, thick, white, opake Subſtance, called the *Sclerotica*, as BNB. Within this is another thin, ſoft, and blackiſh Tunic, called the *Choroides*; which ſerves as it were for a Lining to the other, or rather as a delicate *Subſtratum* for the third Tunic called the *Retina*, which is a curious fine Expanſion of the Optic Nerve YZ over all the larger Segment of the Eye, every Way to BB.

481. The leſſer Segment conſiſts of one Coat or Tunic, called the *Cornea*, as reſembling a Piece of tranſparent Horn; this is more convex than the other, and is denoted by BIB. Within this Coat, at a ſmall Diſtance, is placed a circular Diaphragm, as B*o*, B*o*, called the *Uvea*, or *Iris*, becauſe of the different Colours it has in different Eyes. In this is a round Hole in the Middle called the Pupil, as *o o*, which in ſome Creatures is of a different Figure, *viz.* oblong, as in Cows, Cats, *&c.*

482. As the *Cornea* by its Tranſparency admits the Light to enter the Eye, ſo the Pupil is deſtined to regulate the Quantity of the Rays that ought to enter the interior Part of the Eye for rendering Viſion diſtinct, and the Images of Objects properly illumined. To this Purpoſe it is compoſed of two Sets of muſcular Fibres, *viz.* one of a circular Form, which, by corrugating, contract or diminiſh the Pupil; and the other is an *Annulus* of radial Fibres, tending every where

from the Circumference BB of the *Uvea* to the Center of the Pupil, which, by contracting, dilate and enlarge the Pupil of the Eye.

483. Immediately within the *Uvea* is another *Annulus* of radial Fibres, which on the extreme Part is every where connected with the *Cornea*, where it joins the *Sclerotica* at BB; and on the other Circumference it is connected with the anterior Part of the *Capſula* including the Cryſtalline Humour; and is called the *Ligamentum Ciliare*, and ſometimes the *Proceſſus Ciliaris*, and is denoted by B*a*, B*a*.

484. The Bulk or Body of the Eye is made up of three Subſtances, commonly called *Humours*, *viz.* the *Aqueous*, the *Cryſtalline*, and the *Vitreous*. The *Aqueous Humour* is properly ſo called, being every Way like Water, in reſpect of its Conſiſtence, Limpidity, ſpecific Gravity, and refractive Power. It is contained between the *Cornea* and the *Ligamentum Ciliare*, as BIB*aa*B. Thus Humour gives the protuberant Figure to the *Cornea*, which makes the firſt Refraction of the Rays of Light.

485. The ſecond Humour (improperly ſo called) is the *Cryſtalline*, having its Name from reſembling Cryſtal in Clearneſs and Tranſparency. It is denoted by GKHL, and is in Form of a thick Lens unequally convex, whoſe anterior Surface GKH is the Segment of a larger Sphere, and its poſterior Surface GLH the Segment of a leſſer. This Humour is of a ſolid Conſiſtence, and very little exceeds the ſpecific Gravity of Water, *viz.* in Proportion of 11 to 10 nearly, as I have often found by Experiment. It is contained within a moſt delicate Tunic or *Capſula*, called *Arachnoïdes*, every where pellucid as the Cryſtal-

line itſelf. This natural Lens conduces moſt to the Refraction and Convergency of the Rays of Light.

486. The third Humour is the *Vitreous*, (being clear as *Glaſs)* and is largeſt of all in Quantity, filling the whole Orb of the Eye BMB, and giving it a globular Shape. This Humour is exactly like the White of an Egg, and but a little exceeds the ſpecific Gravity and refractive Power of Water.

487. We ſhall now give the Dimenſions of the Eye and its ſeveral Parts, (in order for Calculation) as they have been determined by actual Meaſurement in a great Number of human Eyes with the greateſt Care and Exactneſs. Theſe Meaſures are expreſſed in *Tenths of an Inch*, as follows.

	Tenths.
The Diameter of the Eye from Outſide to Outſide, taken at a Mean from ſix adult Eyes,	IN = 9,4
The Radius of Convexity of the *Cornea*,	BIB = 3,3294
The Radius of Convexity of the anterior Surface of the Cryſtalline, from twenty-ſix Eyes,	GKH = 3,3081
The Radius of Convexity of the hinder Surface, from the ſame Eyes, at a Mean —	GLH = 2,5056
The Thickneſs of the Cryſtalline, from the ſame Eyes,	KL = 1,8525
The Thickneſs of the *Cornea* and Aqueous Humour together,	IK = 1,0358

489. Moreover it is found by Experiment, that the Ratio of Refraction at the *Cornea* I is as

4 to 3, being the ſame with that out of Air into Water; the Ratio of Refraction at K as 13 to 12, and at L as 12 to 13. Theſe Things premiſed, let AX be the Axis of the Eye, and ED a Ray parallel thereto, and incident on the *Cornea* very near it at D; we are to determine the *Foci* of the ſeveral Refractions of this Ray at the ſeveral Surfaces I, K, and L.

489. The firſt Focus is determined by the Theorem (400) $\frac{mdr}{md - nd - nr}$; for ſuppoſing all behind the *Cornea* BIB were the aqueous Humour continued, then ſince in this Caſe $m = 4$, $n = 3$, $r = 3{,}3294$, and d is infinite, we have $f = 4r = 13{,}3176 =$ IQ, the focal Diſtance from I by the firſt Refraction.

490. The Ray tending by this Means from D to Q, falls converging on the anterior Surface of the Cryſtalline Humour at S. We muſt now find the Focus of the converging Ray DS refracted through a Medium every where the ſame with the Cryſtalline Humour. This we do by the ſame Theorem; for at K we have $m : n :: 13 : 12$, and $r = 3{,}308$, and IQ — IK $= 12{,}2818 =$ KQ $= d$, the Diſtance of the Radiant Q from the Point K; but d is in this Caſe negative, or $-d$, and the Theorem is $\frac{-mdr}{-md + dn - nr} = f = 10{,}06 =$ KP, the new focal Diſtance from K, by the ſecond Refraction.

491. The Ray converging from S to P is intercepted by the hinder Surface of the Cryſtalline at T, and meeting there with a Medium of different Denſity, and a concave Surface, is again refracted by it; and here we have $m : n :: 12 : 13$, alſo the Radius r is negative, as well as d;

and here it is $-r = 2,5056$, and $-d =$ KP $-$ KL $=$ LP $= 8,31$; therefore the Theorem is $\frac{mdr}{-md+nd+nr} = \frac{12\,dr}{d+13\,r} = f =$ LM $=$ 6,112, the laſt focal Diſtance required.

492. The Point M therefore is that in which parallel Rays ED are collected within the Eye, and where the Images of the remote Objects are formed. The Diſtance of this Point from the *Cornea* is IM $=$ IK $+$ KL $+$ LM $=$ $1,036 + 1,852 + 6,112 = 9$. Then IN $-$ IM $= 9,4 - 9 = 0,4 =$ NM. Now the Thickneſs of the *Sclerotica* is by the Micrometer found to be very nearly 0,25; then $0,4 - 0,25 = 0,15$; which is much about equal to the Thickneſs of the *Choroïdes* and *Retina* together. *Hence we ſee the Forms and refractive Power of thoſe ſeveral Humours are ſuch as nicely converge parallel Rays to a Focus upon the* Retina *in the Bottom of the Eye.* And from hence it follows, *that ſince parallel Rays only have their Focus upon the* Retina, *they alone can paint an Image there diſtinctly, or produce diſtinct Viſion of an Object.*

CHAP.

CHAP. II.

The THEORY of *Viſual* GLASSES in reſpect of *long* and *ſhort-ſighted* EYES; and for the Uſe of DIVERS, to ſee under Water.

493.

FROM the foregoing Account of the Eye, it is evident that no longer than the Eye has its juſt Degree of Convexity in the *Cornea*, can the Image of diſtant Objects be perfectly formed upon the *Retina*, and conſequently no longer can there be any diſtinct Viſion effected by the Eye. Now this requiſite Figure of the *Cornea* is liable to be altered ſeveral Ways, but of theſe, two are moſt general, *viz.* by *Age* and by *Accident*.

494. When People grow in Years, the Decays of Nature become gradually ſenſible, and among others, the *Aqueous Humour* begins to be deficient, and the Conſequence is that it is thereby rendered not ſo protuberant or convex as it uſed, and ought, to be. As the Eye becomes leſs convex, its focal Diſtance is enlarged; and the Rays from near Objects now tend to a Point or Focus beyond the *Retina*; therefore upon the *Retina* no perfect Image can be depicted in ſuch Eyes, and ſo no diſtinct Viſion is poſſible.

495. On the other Hand, it often happens that the *Cornea* is *too convex*, and the Rays of

Light from diſtant Objects are converged to Focus before they arrive at the *Retina*; and ſo of Courſe the Image upon the *Retina* muſt be very imperfect, and Viſion indiſtinct in ſuch Eyes. Now Glaſſes may be formed convex or concave that will in a great Meaſure ſupply what is wanting in the Figure of the Eye, and thereby prove the happy Remedy of ſuch Defects.

496. A Perſon long-ſighted by Age muſt have a convex Glaſs placed before his Eye to ſupply the Want of Convexity in the *Cornea*, and to know what Degree (or Radius) of Convexity will exactly ſuit the Eye, the following Method is to be taken. Let BG be the Axis of the Lens AD, whoſe focal Diſtance AO is required. Then let the neareſt Diſtance at which the Eye at A can ſee diſtinctly without a Glaſs be AB: and ſuppoſe C be the Point he would endeavour to ſee or to read at in a nearer Diſtance AC; then the Rule to find the Radius AO is this; let it made as BC to CA, ſo is CA to CO; and that will give the Point O, or Radius AO required. *Fig.* IV. *Plate* VI.

497. For 'tis evident ſince the Eye can ſee by the convering Rays ABD or GBE, if a Lens be interpoſed, ſuch as will cauſe all the Rays CDF after Refraction to paſs to the Eye in the Direction DE, it will produce the ſame diſtinct Appearance of the Point C as the naked Eye has of the Point B, the Proof of the above Rule is as follows; put AB $= d =$ diſtance to which the converging Rays ED tend; let AC $= f =$ Focus correſponding to the Point B, and to which the Rays ED are converged by the Lens AD; and let AO $= r =$ Radius; then by the Rule we have, $d - f : f :: f : r - f$; which

gives $dr - df - rf + ff = ff$, that is, $dr = df + rf$, and ſo $\frac{dr}{d+r} = f$, which is the Theorem for *converging Rays* DE, as at Article (441.)

498. To exemplify this Rule; ſuppoſe you can ſee diſtinctly to read a common ſized Print at no leſs Diſtance than 3 Feet, and you want to read or write at the Diſtance of 12 Inches; then is AB = 36, AC = 12, and BC = 24; therefore by the Rule ſay as 24 : 12 : : 12 : 6 = CO. Then 6 + 12 = 18 = AC the focal Diſtance of the Lens that will procure diſtinct Viſion to the Eye at A.

Fig. V.

499. On the other Hand, for the *Myops* or ſhort-ſighted Perſon, 'tis eaſy to find a Glaſs that ſhall give him a diſtinct View of diſtant Objects. Thus let him determine the fartheſt Diſtance AO, at which an Object appears diſtinct to the naked Eye; then will that Diſtance AO be the Radius of Concavity for the Lens AD as required. For all Rays BD parallel to the Axis AO (and ſuch are Rays from diſtant Objects,) will after Refraction through the Lens AD proceed diverging to E, as if they came from the Center of Concavity O, and, therefore, will of Courſe produce diſtinct Viſion to the Eye at A. The Lens AD is ſuppoſed to be equally concave on both Sides. All this Affair is evident from the Theorem in Article (438.)

Fig. VI.

500. If it be required to fit the Eyes of a *Diver* (or any Perſon under Water) with Glaſſes to ſhew diſtant Objects diſtinctly through that Element, you proceed thus; it was ſhewn that Rays ED parallel to the Axis of the Eye, were refracted towards a Point Q in the Axis by the

aqueous Humour at the *Cornea* in D (489). But when the Eye is under *Water*, the Medium on both Sides the *Cornea* is the ſame, and therefore the Rays DE ſuffer no Refraction at all. In this Caſe a Glaſs Lens, (*Plano Convex*) DB, placed with its convex Side towards the Eye, muſt be ſuch as will converge parallel Rays to the Point Q, ſo that IQ = 1,332.

Air
3 to 2 Glass
4 - 3 Water

501. In order to this it muſt be obſerved, that the Ratio of Refraction out of Air into Glaſs, is that of 3 to 2, but out of Air into Water, it is as 4 to 3 (205), therefore the Ratio of Refraction out of Water into Glaſs is as 3 × 3 to 4 × 2, or as 9 to 8; whence $\frac{m-n}{n} = a = \frac{1}{8}$. And for parallel Rays we had $\frac{r}{a} = f$ (435), or $r = \frac{1}{8}f = 1{,}66$ Tenths of an Inch, the Radius of the Plano-Convex as required.

502. If it be deſired to have the Lens convex on both Sides, and equally ſo, the Radius of the Lens muſt be equal to that of the *Cornea*, for let the latter be called R, and the former r; then by Refraction at the *Cornea* only, we have $4\,R = f =$ IQ, (400). But under Water, with the double convex Glaſs, we have $\frac{r}{2a} = f = 4\,r =$ IQ (435.) Therefore $4\,R = f = 4\,r$, that is, $R = r$. Q. E. D.

N. B. Since the Lens BD converges the Rays to Q, the ſeveral Humours of the Eye will converge them yet more, and unite them upon the *Retina* at M, as in common Viſion.

CHAP. III.

The THEORY *of* Refracting TELESCOPES *explained.*

503.

LET AC be the Object Glaſs, and DP the Eye Glaſs of a Teleſcope, whoſe Axis CP enters the Eye at T, and falls upon the Optic Nerve in the Bottom of the Eye at N. Alſo let the focal Diſtance of the Object Lens be CF, and that of the Eye Glaſs be PF; laſtly, let the Semi-aperture of the Teleſcope be AC; and ſuppoſe every Thing ordered in the *beſt Manner in Regard to Light and Diſtinctneſs of Viſion.* *Plate* VI. *Fig.* VII.

504. Now let it be propoſed to conſtruct another Teleſcope of a greater Length and Power of magnifying, whoſe Object Lens *a c* has its focal Diſtance *c f* in a given Ratio to CF, and let the Diameter of the Aperture *a a*, and focal Diſtance *p f* of the ocular Lens be required, ſuch, that this large Teleſcope may ſhew the Object *equally bright and diſtinct*, as the ſhorter one with the Glaſſes AC, and DP. *Fig.* VIII.

505. Let all the correſponding Parts in each Conſtruction be denoted by the ſame Letters, in

one large, in the other ſmall; and the Parts in one perfectly ſimilar to thoſe of the other.

506. Then if HA be a Ray falling upon the Object Lens AC parallel to the Axis CT, it will be refracted into the coloured Beam OAD; in which the Red Ray croſſes the Axis in the Point F, and the violet Ray croſſes it in the Point B. On the other Hand, let ED be a Ray incident upon the ocular Lens DP, parallel to the Axis TC, and ſuch as will be refracted into the coloured Beam FDG, ſo that the red Ray DF in this Beam, and AF in the other, may croſs the Axis in the ſame Point F, and that F the Focus of the red Rays, may be the ſame in both, or the common Focus of the Lenſes AC and DP.

507. Further it is ſuppoſed, that the Conſtitution of the Eye is ſuch, that it will converge the red Rays which it receives in a Direction parallel to the Axis TN, to one Point upon the *Retina.* And that the Aberration from the *ſpherical Figure* of the Lenſes is of no Moment compared with that ariſing from the *different Refrangibility* of the Rays.

508. Then ſince the violet Ray GD is refracted by the Lens DP into the Ray DE parallel to the Axis PN, it is plain the violet Ray ABD proceeding from the Point B in the Axis, cannot be refracted into DE, but will be carried within it, towards the Pupil T, in ſome other Direction DK, ſo as to make the Angle EDK equal to the Angle BDG (454).

509. Draw KO parallel to the Axis, and it will be a red Ray AFO, refracted through the Lens DP to the Eye. Then ſince only FD, FO, can (being *red Rays*) be refracted to the Eye in Directions DE, and OK, parallel to the Axis CN, theſe

alone can be refracted to one and the ſame Point N in the Bottom of the Eye. Therefore the Ray DK even though it were a red one, could not be refracted to the Point N, much leſs can it as being of a Violet Colour. The Ray DK muſt therefore be refracted to ſome other Point M within the Ray KN, making the Angle NKM in a conſtant Ratio to the Angle DKO (390.)

510. Now ſince in the original Ray AH, the red Part AO is refracted to the Point N upon the *Retina*, and the Violet Part AD is refracted to the Point M, therefore the Line NM will be the Diameter of the Circle into which the Rays AH will be diſſipated, and which is therefore called the *Circle of Aberration*.

511. Since it is required that Viſion ſhould be equally diſtinct in both Teleſcopes, 'tis plain we muſt in each have $NM = nm$, or the Angle $NKM = nkm$; and therefore the Angle OKD, or $EDK = edk$, or $BDG = bdg$; laſtly, ſince the Angles FDG, fdg, are very ſmall, compared with BDF, and bdf, and though not preciſely equal, yet may be taken as ſuch without ſenſible Error, therefore we ſhall have the Angle $BDF = bdf$.

512. In the firſt Teleſcope put $AC = a$, $CF = b$, $FP = c$; and in the latter, make $cf = d$, $pf = y$, and $ac = x$; and laſtly let it be made as CF to FP, ſo the Line Tf to pf; and then will $Tf = \frac{by}{c}$. *Fig.* VIII.

513. Let FS be an Image formed by the Object Lens AC of any diſtant Object by Rays IC, RC, coming from its extreme Parts, and interſecting each other in the Center C of the Lens. Then as theſe Rays are the Axes of two Pencils

of Rays emitted from each extreme Part of the Object, it is evident, *That the Angle under which the Object and its Image are ſeen from the Center* C *of the Object Glaſs are the ſame.*

514. Again, ſince all the parallel Rays which fall upon the Glaſs between C and A are converged to a Point F in the Image; it is again evident, *That every Point in the Image, and conſequently the whole Image, will be illumined in Proportion to the Area or Aperture of the Object Glaſs, or to the Square of its Diameter, or Semidiameter* AC.

515. Draw the Line PS, then the Angle FPS is that which the Image FS ſubtends at the Eye-Glaſs DP, and is to the Angle FCS, under which the Image is ſeen from the Object Glaſs AC, nearly as EC to FP; *which Ratio of* FC *to* FP *will therefore expreſs the magnifying Power of the Teleſcope.*

516. Theſe Things premiſed, in order *that both the Teleſcopes may ſhew Objects equally luminous,* it is neceſſary that the Apertures of the Object Glaſſes ſhould encreaſe with the Areas of the Pictures, or that $AC : ac :: FS : fs :: FC$ $(= fT) : fc$; which is in Symbols, $a : x :: \frac{by}{c} : d$ (512). Whence we have $pf = y = \frac{abc}{dx}$.

517. Draw FQ perpendicular to the Axis CP and cutting the Ray AD in Q. Then becauſe $CB : BF :: cb : bf$ (463) we ſhall have $CA : FQ :: ca : fq$. And becauſe it is required *that Objects in both Teleſcopes ſhould appear with equal Diſtinctneſs,* the Angles of Aberration BDF and bdf muſt be equal, and we ſhall have $DF : FQ :: df : fq$; or $PF : FQ :: pf : fq$, becauſe of the very ſmall Difference of PF and DF, in

either Conſtruction. Therefore it will be PF : pf : : FQ : fq : : CA : ca; that is, $c : \frac{adc}{bx} :: a : x$. Whence $bxx = aad$, and conſequently it is $x^2 : a^2 :: d : b$. Or $x : a :: \sqrt{d} : \sqrt{b}$. That is, *The Diameters of the Apertures of the Object Glaſſes are in the ſubduplicate Ratio of their Focal Diſtances.*

518. But *the Focal Diſtances of the ocular Lenſes* FP, fp, *are in the ſame direct Ratio of the Diameters of the Apertures*; for we had (516) $a : x :: \frac{by}{c} : d$; therefore $aa : xx :: \frac{bbyy}{cc} : dd$; but we had $aa : xx :: b : d$ (517), therefore $\frac{bbyy}{cc} : dd :: b : d$. From thence we get $byy = ccd$; whence $cc : yy :: b : d :: aa : xx$; that is $c : y :: a : x$. Or FP : fp : AC : ac. Q. E. D.

519. Hence it appears, that in two ſuch Teleſcopes, *the focal Diſtances of the ocular Lenſes are in the ſubduplicate Ratio of the focal Diſtances of the Object Lenſes*; for we had $a : x :: \sqrt{b} : \sqrt{d}$ (517) $:: c : y$ (518) that is, PF : pf :: $\sqrt{CF} : \sqrt{cf}$.

520. Put the magnifying Power in the Short Teleſcope $m = \frac{b}{c}$, and that of the long one M $= \frac{d}{y}$; then we have $mc = b$, and $My = d$; whence $b : d :: mc : My :: aa : xx$; which will give $mcxx = Myaa$; hence $c : y :: Maa : mxx :: a : x$ (518); therefore $Mxaa = mxxa$; that is $Ma = mx$; whence $m : M :: a : x$; *Or the magnifying Powers of the Teleſcopes are as the Diameters of their Apertures.*

521. Laſtly, in both Teleſcopes the ſmall Cylynders of Rays, tranſmitted from the Eye-Glaſſes to the Eye, whoſe Semi diameters are OP, *op*, and which contain all the Rays which fall upon the Object Glaſſes AA and *aa* parallel to the Axis, are of the ſame Magnitude; that is, $OP = op$. For ſince FC : CA : : FP : PO, we have $OP = \frac{ac}{b}$. In like Manner, ſince $fc : ca : : fp : po$, and $fc = d$, and $ca = x$, and it has been ſhewn (516) that $fp = \frac{adc}{bx}$, therefore $po = \frac{ac}{b} = PO$, which was to be ſhewn.

522. If the ocular Lens PD were *concave* inſtead of *convex*, and of an equal focal Diſtance, then all the ſame things might be ſhewn by a ſimilar Demonſtration, which it is not worth while here to repeat, as this Sort of Teleſcopes are but of little Uſe, *having ſo ſmall a Field of View*.

523. As to the other Eye-Glaſſes of a Teleſcope they have been already conſidered in the *firſt Part* of this Work, Page 19, and what is here delivered is the Subſtance of *Hugenius*'s Theory of *Refracting Teleſcopes*. As he has ſaid nothing in regard to the Theory of *Reflecting* TELESCOPES, we ſhall next proceed to that, and give it in as clear and conciſe a Manner as poſſible.

CHAP. IV.

The THEORY of CATOPTRIC, or *Reflecting* TELESCOPES.

524.

AS all the conſtituent Parts of the *Reflecting* TELESCOPE have been already deſcribed in the *third* Part of this Work (ſee Page 88), it remains here only to give the *Rationale* of its admirable Effects as compendiouſly as the mathematical Theory will admit of.

Plate VI. *Fig.* IX.

525. In order to this, let PIC be the Axis of the Teleſcope; HI, TU, the two Eye-Glaſſes; MN, the equivalent Lens; *a a* the large Speculum; AD, the ſmall Speculum; *g*, the Center, and f the *Focus* thereof. Alſo G is the Vertex, and F the *Focus* of the large Speculum. Then the magnifying Power of this Inſtrument is thus computed; let OB be the Image of a very diſtant Object form'd in the Focus F of the large Mirror *a a*. It has been ſhewn (412), that any Object and its Image FB are ſeen under the ſame Angle FGB from the Vertex G of the Speculum. But the Image FB is ſeen from the Vertex C of the ſmall Speculum under the Angle

FCB, and therefore it appears (from what has been ſaid of the Teleſcope (513, 515), that the Object will be magnified by the two Speculums in the Proportion of the Angle FCB to the Angle FGB, or of GF to CF; which Ratio is thus expreſſed $\frac{GF}{CF}$.

526. Again, from the Theory of the *Microſcope* it will appear, that if we conſider the Part FB of the firſt Image as a Microſcopic Object, and removed to a diſtance CF from the little Mirror greater than its focal Diſtance Cf, then a ſecond large Image LV will be formed thereby from the firſt; for a Ray proceeding from the Point F to the ſmall Metal in A will from thence be reflected to the Axis in V in the right Line AV; and a Ray from B paſſing through the focal Point f of the ſmall Mirror to the Point A will from thence be reflected parallel to the Axis in the right Line AL; and conſequently LV will be the magnified Part FB in the firſt Image. And becauſe the Lens MN is placed at its focal Diſtance PV from that ſecond Image LV it will ſhew it very diſtinctly, and in Compariſon of the firſt Image FB it will be magnified in the Ratio of CV to PV, (which is thus expreſſed $\frac{CV}{PV}$,) becauſe in that Ratio are the optic Angles LPV and LAV.

527. Therefore the whole magnifying Power of a reflecting Teleſcope will be expreſſed by the compound Ratio $\frac{GF}{CF} \times \frac{CV}{PV}$. In this Expreſſion the Parts GF and PV are known as being the focal Diſtances of the great Metal, and Eye-Glaſs MN. Alſo ſome Parts of the Line CV are known, as GF, and GI taken at Pleaſure. From thence

we can find V I, by conſidering V as the Point of Diſtance to the *converging Rays* AK, which by the Lens HI are refracted to the Focus Z; therefore by the Theorem (441) $\frac{dr}{d+r} = f$, we have $d = \frac{rf}{r-f}$; but $r = \mathrm{F} =$ the Focal Diſtance of the Lens HI; and if we put D = IT = Diſtance between the two ocular Lenſes, and TZ = y the focal Diſtance of the Lens TU, then IZ = $f = \mathrm{D} - y$, and conſequently $d = \frac{\mathrm{F} \times \overline{\mathrm{D} - y}}{\mathrm{F} + y - \mathrm{D}}$ = IV, which is therefore known; and therefore alſo the Line or Diſtance FV is known.

528. It remains now only to find the Part CF, which is thus inveſtigated; put FV = G, C f = $\frac{1}{2}r$, CF = d, CV = f; and by the Catoptric Theorem $\frac{dr}{2d-r} = f$ (405) = G + d, we get $2\mathrm{G}d + 2d^2 - \mathrm{G}r - dr = dr$; and by making $\mathrm{G} - r = 2b$, and extracting the Square Root, we have $d = \sqrt{\frac{1}{2}\mathrm{G}r + b^2} - b$ = CF; whence CF + FV = CV, the Quantity ſought.

529. The Angle of Viſion T Q U is thus known; ſince LV = HI = half the Aperture of the large Lens, ſay, As VP to VL ſo is the Radius to the Tangent of the Angle LPV = TQU, becauſe QU is parallel to PL, as is evident by conſidering that the Angle LPV = 2 UPT = TQU, becauſe TP = 2 TQ (116.)

530. Again, ſince TX is alſo parallel to PL, if we ſay, as Radius to the Tangent of the Angle of Viſion XTZ ſo in TZ (the Focus of the ocular Lens) ZX, the Semi-diameter of the *Field of View* and conſequently of the Hole of Diaphragm, or Eye-ſtop.

531. Laſtly, we have CV : IV : : AC : IK, which is therefore known; then IZ : IK : : TZ : TS, the Semi-diameter of the Cylindrule of Rays which enter the Pupil of the Eye, and is therefore the Meaſure of the ſmall Hole in the End of the Piece next the Eye.

532. Theſe are all the eſſential Particulars relative to the due Conſtruction of a *Reflecting* TELESCOPE, which we ſhall now illuſtrate by Computation in one of the common Size, *viz.* of a *Foot Length*. In ſuch an one the focal Length of the great Speculum is GF = 9 Inches; that of the ſmall one $Cf = 1{,}5 = \frac{1}{2}r$. The focal Diſtance of the Lens IH is IP = 3; of the occular Lens TU, is TP = 1; and the Diſtance between them IT = 2. Alſo the focal Diſtance PV = IV = 1,5; and the Diſtance IG = 0,5. Thence it is VG = 2; FV = 11; and $r = 3$; therefore $FV - r = G - r = 8$; whence $b = 4$, and $b^2 = 16$; conſequently $\sqrt{\frac{1}{2}Gr + b^2} - b = CF = 1{,}709$.

533. Then CV = 12,7; and ſo the magnifying Power $\frac{GF}{CF} \times \frac{CV}{PV} = \frac{9 \times 12{,}7}{1{,}7 \times 1{,}5} = 44{,}6$. That is, the Object is by ſuch a Teleſcope magnified 44,6 times.

534. If the occular Lenſes be ſuch as give the focal Diſtance PV = 1,25; then ſay as 1,25 : 1,5 : : 44,6 : 53,5 nearly, the magnifying Power in this Caſe.

535. If the focal Diſtance of the ſmall Speculum be Cf = 1,25, then ſay, as 1,25 : 1,5 : : 53,5 : 64, nearly; or it magnifies an Object 64 times, in its linear Dimenſions.

536. In the ſame Manner you proceed to calculate the magnifying Power of any larger Teleſcope. But before I leave this Subject I ſhall ſhew how the Equation $\sqrt{\frac{1}{2}Gr + b^2} - b =$ CF, may be conſtructed geometrically, and conſequently the required Line CF found readily by the *Plain Scale and Compaſſes.*

537. Draw any right Line AD of a ſufficient Length, in which take a Point B, and from thence ſet off the Diſtance BA $= \frac{1}{2}$ FV; alſo in the ſame Line make BD $= r$, the Radius of the little Speculum; then biſect AD in C; upon the Point C, with the Radius CA, deſcribe the Semicircle AED. Upon the Point B erect the Perpendicular BE, which will cut the Circle in E; and from B ſet off the Diſtance equal to $\frac{1}{2}$ FV $- \frac{1}{2} r$ ($=$ H) to F; and with the Extent FE deſcribe the Arch EG; then will BG be equal to FC the Diſtance required from the ſmall Speculum. The Truth of this Proceſs is known to every Mathematician. *Plate* VII. *Fig.* I.

CHAP. V.

The METHOD of making the *Reflecting* TELESCOPE an *Univerſal* PERSPECTIVE for viewing all Kinds of OBJECTS.

538.

FROM the foregoing Theory of the *Reflecting* TELESCOPE, it appears that the ſame Inſtrument is capable of ſerving many other Purpoſes in Optics, not uſually conſidered, (or rather not at all,) though nevertheleſs curious and entertaining. For this TELESCOPE is at the ſame time an ENGISCOPE, a MEGALASCOPE, and a *Catoptric* MICROSCOPE of two Sorts, as will thus appear:

I. *An* ENGISCOPE.

539. By the Word ENGISCOPE is to be underſtood an Inſtrument that will ſhew any Object at a *near Diſtance*, and magnify them to the View. The Theory of the Teleſcope depends upon this Conſideration, that in the general *Catoptric Theorem* (405) $\frac{dr}{2d-r} = f$, the Diſtance d of the Object being ſuppoſed infinite, the focal Diſtance becomes is that Caſe, *half the Radius of the Speculum*. But it is plain if the Diſtance of

the Object is not very great, then all the Terms in Theorem muſt be employed to aſcertain the focal Diſtance *f*, which will now exceed the ſolar focal Diſtance GF, and the more ſo as the Diſtance *d* of the Object is leſs.

540. Therefore if we ſuppoſe 15 Feet (the general Width of Rooms) to be the neareſt Limit for the Uſe of this Inſtrument as an ENGISCOPE, we have by this Theorem, the focal Diſtance GF of the great Speculum *a a*; for the Radius is $r = 18$ Inches, and the Diſtance d (= 15 Feet) = 180, we have the focal Diſtance $f = \frac{3240}{342} = 9{,}5$ Inches, nearly.

541. Having thus obtained the focal Diſtance GF for an Object diſtant 15 Feet, you find how much it is magnified by the very ſame Proceſs as before (528) which therefore needs not to be here repeated. It may only be obſerved that there is generally Room enough in Reflectors to remove the ſmall Metal ſo far from the large one as is ſufficient to anſwer this Purpoſe.

II. *A* MEGALASCOPE.

542. The Uſe of the *Reflecting Teleſcope* as a MEGALASCOPE has not been conſidered by any Writer on Optics that I know of, though it be entertaining and curious to every one who is deſirous of any Improvement of their Sight. By a MEGALASCOPE is meant ſuch an Inſtrument as *ſhews any of the larger Sort of ſmall Objects in a magnified View*; and this Effect by the Teleſcope is much preferable to that by a common *Magnifier*, or thoſe called *Cloth* or *Flower Microſcopes*; as in them you are obliged to bring the

Object near the Eye, but in theſe *Teleſcopic* MEGALASCOPES, you view every Object in the Place where it is, without removing or diſturbing it, by which Means many curious Subjects both in Art and Nature are eaſily viewed and ſurprizingly magnified, which would otherwiſe paſs unobſerved by us.

543. All this will be ſufficiently manifeſt by Calculation. Any Object may be eaſily and diſtinctly ſeen in the Reflector when placed at *four times the ſolar focal Diſtance from the Metal.* Therefore when $GF = 9$ Inches $= \frac{1}{2}R$, and $d = (4GF =)\ 36$; we ſhall find $\frac{dR}{2d - R} = f =$ 12 Inches, which will be the Diſtance of the Image OB from the Speculum *a a.* Then is $FV = 14$; alſo $FV - r = 11$; whence $b = 5\frac{1}{2}$; and ſo $\sqrt{\frac{1}{2}Gr + b^2} - b = CF = 1{,}66$. Then $CV = 15{,}66$; and ſo the manifying Power will be, in this Caſe, $\frac{12 \times 15{,}66}{1{,}5 \times 1{,}66} = 75\frac{1}{2}$.

544. But if the focal Diſtance PV of the equivalent Lens NM, and that of the ſmall Metal Cf be each $1\frac{1}{4}$ Inch, the magnifying Power will be 104; or any Object placed 36 Inches from the great Speculum will be magnified 104 times. You will eaſily judge how wonderfully different any Object muſt appear when magnified to ſuch a prodigious Degree.

III. *A* CATOPTRIC MICROSCOPE *by two Reflections.*

545. The *Reflecting* TELESCOPE is a *Catoptric* MICROSCOPE on a double Account, that is, by forming *two Images*, or *one only*, of the Object

propoſed; for when any Object is magnified 10, 15, or 20 times in its linear Dimenſions by any Glaſs or Inſtrument, then ſuch a Glaſs may be properly called a MICROSCOPE, becauſe a ſmall Object then becomes ſufficiently magnified to view all its Parts diſtinctly through it.

546. HUGENIUS ſuppoſes the neareſt Limit for diſtinct Viſion (to Eyes in general) is *eight Inches*. Then 'tis plain any ſmall Object placed at eight Inches from the Eye, and then at 36 Inches Diſtance, will ſubtend Angles which will be inverſely as thoſe Diſtances, or as 36 to 8; but an Object at 36 Inches viewed through the Teleſcope, ſubtends an Angle 104 times larger than the Angle it ſubtends to the naked Eye at that Diſtance. Therefore by compounding theſe Ratios, we have the Ratio of 832 to 36, or of 23 to 1, to expreſs how much the Angle is greater through the Teleſcope at the Diſtance of 36 Inches, than is ſubtended at the naked Eye at the Diſtance of 8 Inches.

IV. *A* Catoptric MICROSCOPE *by a ſingle Reflection.*

547. If OB be no longer conſidered as an Image of an Object, but a real Object itſelf, of the *microſcopic* Kind, then will an Image LW be formed thereof by Reflection from the ſmall Metal AD, which viewed by the ſingle Eye-Glaſs MN, the magniſying Power will be as great as in a *dioptric Microſcope* with the ſame Eye-Glaſs, and the focal Diſtance of the ſmall Object Lens, the ſame as that of the ſmall Metal here. But Viſion will be more diſtinct in this on a double Account, *viz.* (1.) becauſe there is

no

no Aberration of Rays in the Focus, ariſing from their *Colorific Quality*; and, (2.) becauſe the Aberration cauſed by the Figure of the Metal is far leſs than that occaſioned by the Figure of the Lens. Whence ſuch a *Catoptric* MICROSCOPE muſt be eſteemed the moſt perfect of all others of the compound Sort.

V. *A* HELIOSCOPE.

548. The Word HELIOSCOPE denotes an Inſtrument by which the *Sun may be viewed, without Offence to the Eyes.* Now if OB be the Diameter of the Sun's Image formed by the large Metal *a a*; then, having taking off the Eye-Piece, a ſecond large Image whoſe Diameter is LW will be formed from the firſt by the ſmall Speculum AD. And this large ſecondary Image will be ſo very perfect, that you may ſee not only the *Maculæ* or black Spots, but even the *Feculæ* or bright Spots upon the Sun's Diſk; and this either with the naked Eye, or by a Magnifying Glaſs. There are other Methods of making a HELIOSCOPE of this Inſtrument, but as they are leſs perfect, I ſhall not ſtay to deſcribe them here.

VI. *A* Solar MICROSCOPE.

549. There is no Inſtrument which ſo properly deſerves the Title of the *Solar Microſcope* as that which is made of the *Reflecting* TELESCOPE; for a large Image of the Sun being formed at LW, the *Microſcopic Objects* are placed in that very Image, and are magnified as

uſual by a ſmall Lens US, but the Solar Image being magnified at the ſame, affords a moſt glorious Spectacle of the magnified Images of Objects upon the Face or Diſk of the Sun itſelf; by which Means (without darkening the Room) the Pictures of ſuch Objects are drawn with the greateſt Eaſe and Exactneſs, in different Sizes at Pleaſure. And I am thoroughly perſuaded if this Method of making a genuine Solar Microſcope were known more generally, the common Sort would ſoon come into Diſuſe; at leaſt this only would pleaſe the more curious and diſcerning Part of Mankind.

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CHAP. VI.

The THEORY *of* Single MICROSCOPES conſtructed with LENSES or SPHERULES.

550.

WITH regard to the THEORY of MICROSCOPES, ſingle and double, as well as all other optical Inſtruments, the principal Thing to be conſidered is the Image of an Object upon the Retina in the Fund of the Eye, becauſe it is by Means of this Image the Object itſelf is repreſented to the Mind under all the various Qualities of Magnitude, Poſition, Diſtinctneſs, Brightneſs, Diſtance, Colour, Motion, *&c.*

551. In *ſingle* MICORSCOPES, which are made of ſingle *Lenſes* or *Spherules*, the *magnifying Power* is aſcertained by comparing the Quantity of the optic Angle under which the Object appears through the Lens with that under which it appears to the naked Eye at the Diſtance of 8 Inches, ſuppoſing that to be the general Limit of diſtinct Viſion. It has been ſhewn (35) that the Lens produces diſtinct Viſion of an Object placed in its Focus by refracting the Rays to the Eye, parallel to the Axis thereof; and conſequently the Object being ſeen clearly at the Diſtance of the Focus of the Lens, will there ſubtend an Angle at the Eye as much greater than it does at 8 Inches diſtance from it, as 8 Inches is greater than the focal Diſtance of the Lens or Spherule. *Therefore the magnifying Power (for the linear Dimenſions) will be inverſely as the focal Diſtance of the Lens.*

Plate VII. *Fig.* II. 552. If a Spherule be uſed in magnifying ſmall Objects inſtead of a Lens, (as they often are, being eaſier made) then the Power of magnifying in a Lens and Spherule being equal, the Lens will have one great Advantage over the Spherule, *viz. that there* will be 3 times the Diſtance between it and the Object, as there is between the Object and the Spherule, which is thus ſhewn. Let LEN be a Lens, and EC its focal Diſtance; then with the Radius AE $= \frac{2}{3}$ EC, deſcribe a Circle ADGF to repreſent the Spherule, whoſe magnifying Power will be equal to that of the Lens, and its focal Diſtance will be AC $= \frac{1}{2}$ AE, by the Theorem at (430). Therefore the Diſtance EC of the Object from the Lens is equal to three times the Diſtance AC of the ſame Object from the Spherule.

553. In what has been hitherto ſaid, we have not conſidered the Thickneſs of the Lens; but when the Lens is to magnify much with a very ſmall focal Diſtance, its Thickneſs muſt be conſiderable; ſo as often to equal the *third Part of a Circle* in its convex Segments; and this is generally ſufficient for the deepeſt Magnifiers.

554. Therefore ſuppoſe in the Lens LEN the Segment LIN = LHN = third Part of a Circle LMN; in this Caſe the Thickneſs of the Lens IH will be equal to the Radius; and the focal Diſtance of parallel Rays after Refraction at the firſt Surface LIN will be = 3 IH, or 3 times the Radius (429) therefore in Theorem of (425) if we make $t = r$, and $f = 3r$, that Theorem will become $\frac{4}{5} r$ = f = Hf = the Diſtance of the focus f from the Surface of the Lens at H.

Fig. III.

555. Let a Spherule now be ſuppoſed to magnify juſt as much as that Lens, and denoted by the Circle DGF, deſcribed upon the Center E of the Lens; and ſince, by Suppoſition, the optic Angle OEB or the Diſtance EC (= Ef) is the ſame in both the Lens and Sphere, and becauſe the focal Diſtance Gf is ever = $\frac{1}{2}$ Radius EG, we have the Proportion of the ſaid Radius EG of the Spherule to the radius HI of the Lens thus eſtimated. We have ſhewn that Hf = $\frac{4}{5}$ HI; therefore EF = $\frac{13}{10}$ HI; but EG = $\frac{2}{3}$ Ef = $\frac{26}{30}$ HI; conſequently HI : EG : : 30 : 26 : 15 : 13 : Radius of the Lens : the Radius of the Spherule.

556. Then from Ef take EG, there will remain Gf = AC = $\frac{13}{30}$ HI, but IC is $\frac{4}{5}$ HI; therefore the Diſtance of the Object IC from the Lens

is to the Diſtance from the Spherule AC as $\frac{4}{5}$ to $\frac{13}{30}$; that is, as 2 to 1 very nearly. Whence appears the great Advantage of a Lens over a Spherule of the ſame magnifying Power.

557. What regards the *Diſtinctneſs of Viſion* in two different Lenſes, may be explained as follows; Let P be a Lens whoſe Axis is TF, *Fig.* IV. & V. and PD the Semi-diameter of its Aperture, the greateſt it will bear, which is ſuppoſed leſs than the Semi-diameter ET of the Pupil of the Eye. And let ED be an extreme Ray parallel to the Axis TF, the red Part of which is refracted to the Focus F, and the violet Part to B. And let all the ſame Things be denoted by ſmall Letters in a ſmaller Lens *p*, whoſe Semi-aperture *pd* is to its focal Diſtance *pf* in the ſame Ratio as in the larger one.

558. In two ſuch Lenſes *Viſion will be equally diſtinct*; for the parallel Ray ED is diſſipated through the Angle FDB; and on the other Side a Ray FD coming from a Point F in an Object will be diſſipated through the Angle EDK, in ſuch Manner that the Angle EDK will be equal to the Angle FDB, as was ſhewn (454). But the Angle FDB is that on which the Aberration in the Fund of the Eye depends (510). Therefore becauſe from the Nature of this Aberration, we have PF : FB : : *pf* : *fb*; and by Hypotheſis, it is PD : PF : : *pd* : *pf*; it is manifeſt that the Angles PDF and *pdf* are equal, as alſo the Angles PBD and *pbd*; and that therefore the Difference of the former will be equal to the Difference of the latter, that is, the Angle FDB = *fdb*, or EDK = *edk*; and conſequently the Aberration in the Eye will be the ſame in both, and of Courſe the Viſion will be *equally diſtinct*.

559. Laſtly, in regard to Light, and the apparent Brightneſs of Objects ſeen through theſe Lenſes, we may obſerve, that becauſe the Angles PFD and *pfd* are equal, the ſame quantity of Light will enter the Eye through each Lens from the ſeveral Points of an Object; but ſince the Picture of Objects upon the *Retina* made by the Lenſes, are as the Squares of the focal Diſtances PF and *pf*, (551) and ſince the ſame quantity of Light illuminates both Pictures, 'tis evident the Brightneſs of the Picture (or of the Object) by the Lens P will be to that of the Lens *p*, as PF^2 to pf^2.

560. Hence it follows, that ſince we can't proceed to magnify Objects by ſmall Lenſes with equal Diſtinctneſs as in large Ones, without encreaſing the Obſcurity of the View, it is certain our Progreſs in magnifying by ſingle Lenſes is thereby limited, unleſs we enlighten the Objects themſelves in Proportion as they are magnified in their Surfaces by theſe ſmall Lenſes.

561. But ſuppoſing we could do this, yet then the Advantage would not be very extenſive, as it is found by Experience, that the Viſion of Objects becomes imperfect by the Smallneſs of the Cylindrule of Rays from the minute Apertures of ſuch very ſmall Lenſes. HUGENIUS fixes the utmoſt Limit of the Aperture to the fifth or ſixth Part of a Line, but from late Experiments we may allow it to be $\frac{1}{100}$ Part of an Inch.

562. Before we drop the Subject of ſingle Microſcopes, it will be proper to exemplify their Power of magnifying Objects, which we ſhall do in a Lens of $\frac{1}{10}$ of an Inch focal Diſtance. By ſuch a Lens an Object is ſeen diſtinctly at $\frac{1}{10}$ of an Inch from the Eye; without it the Object muſt be removed to the Diſtance of 8 Inches

(551) therefore as $\frac{1}{10}$ is to 8, or as 1 to 80, ſo is the Angle the Object ſubtends at 8 Inches Diſtance to the Angle it ſubtends at $\frac{1}{10}$ of an Inch; and conſequently an Object is magnified 80 times by ſuch a Lens, in its *linear Dimenſions*; and 80 times 80, or 6400 times in its *Superficies* or *Area*; and 80 times 6400, or 512000 times, in reſpect of its *Solidity or Bulk.* Thus a Lens $\frac{1}{50}$ of an Inch focal Diſtance will magnify 5 times 80, or 400 times in Length and Breadth. And this is the ſmalleſt Size that Lenſes can well be ground to.

563. Spherules may be made much ſmaller; and we have an account in the *Philoſophical Tranſactions* lately publiſhed, of a Set of Spherules or Glaſſes made by one *Padrè Torrè*, an *Italian*, ſo extremely minute, that the leaſt of them is affirmed not to exceed *half a* French *Point in Diameter*, that is, 0,007416 of an Inch; and ſince the focal Diſtance thereof is $\frac{3}{4}$ of its Diameter (430), it will be only 0,0055625 of an Inch; and therefore by the Rule above, its magnifying Power is as 8 to 0,00556, or as 1438,2 to 1. But this (though prodigiouſly great) is but little more than half what Father *Torrè* makes it, who affirms that it magnifies 2560 times. How this Aſſertion can be ſupported, or made to appear fact, I leave to be ſhewn by thoſe whom it moſt nearly concerns.

CHAP. VII.

An EXPLANATION of the PRINCIPLES upon which the THEORY of *Compound* MICROSCOPES depends.

564.

THE Power of magnifying in ſingle Microſcopes being thus limited, let us next inveſtigate what may be done by compounding two Lenſes together; and here (ſays our illuſtrious Author, HUGENIUS) we ſhall ſhew that much more may be done in ſhort Microſcopes than by long Ones; and that there would be a certain Progreſs of magnifying Objects by theſe ſmall compound Microſcopes even to Infinity, were it not for the Minuteneſs of the Object Lenſes, which renders it difficult to give them a proper Figure, and to adapt them in Inſtruments, eſpecially as by their Tenuity they at length evade the Sight.

565. The THEORY of a *compound* MICROSCOPE conſiſts in the following Particulars. Let PD be the Object Lens, and its focal Diſtance PO; and EZ the ocular Lens, whoſe focal Diſtance is EN. Then let an Object BX be placed

a little farther from the Lens P, than its Focus O, and an Image NY will be formed thereof in the conjugate Focus N, and be viewed by the Eye in the Axis at V. Laſtly, let the ſame Object be viewed by the naked Eye from the Point A; and by taking BQ to BO as EN to OP, the Ratio in which the Object will be magnified, will be that of AB to BQ.

566. For join YE, and parallel to it draw ZV, and XQ. Then the Object appears in the Microſcope under the Angle NEY = EVZ = XQB; to the naked Eye it appears under the Angle XAB; conſequently it will be magnified in the Ratio of the Angles BQX to BAX, or of AB to BQ, by the Principles of Trigonometry.

567. The Triangles PXQ and PYE being ſimilar, give $BQ : NE :: BX : NY :: BP : NP$; but by the common dioptric Theorem $\frac{dr}{d-r} = f$, we have $d : f :: d - r : r$, that is $BP : PN :: BO : PO$; ſo conſequently we have $BQ = \frac{BP \times NE}{NP} = \frac{NE \times BO}{OP}$; therefore the magnified Appearance of the Object will be inverſely as theſe Quantities when AB is given.

568. The Brightneſs or Quantity of Light which illuminates any Particle of the Picture upon the *Retina* in the Eye will be directly as PD^2 when PO is given; and inverſely as PB^2 when PD is given; for in the ſame Lens P, the Quantity of Rays proceeding from any Point B will be proportional to the Area of the Aperture of the Lens, which is as the Square of the Diameter or Semi-diameter thereof, *viz.* PD^2.

569. But becauſe in a Cone of Rays proceeding from a Point B, the Denſity of thoſe Rays decreaſes as the Square of the Diſtance from that Point increaſes, therefore the Denſity of the Rays in the ſame Aperture of different Lenſes will be inverſely as the Square of the focal Diſtance from the Point B, or as $\frac{1}{PB^2}$.

570. Conſequently in different Lenſes, the Brightneſs of any Point in the Image or Picture on the *Retina* will be as $\frac{PD^2}{PB^2}$.

571. The Brightneſs of the whole Picture or Image will be greater in Proportion to the Light of every Part, which is as $\frac{PD^2}{PB^2}$, and leſs in Proportion as the Area of the Picture is greater. Now the linear Dimenſion of the Picture is as $\frac{PN}{BP \times NE}$ (567), the Square of which is $\frac{PN^2}{BP^2 \times NE^2}$; which is therefore as the Area of the Picture. Hence the Brightneſs of the Image will be as $\frac{PD^2}{PB^2} \times \frac{BP^2 \times NE^2}{PN^2} = \frac{PD^2 \times NE^2}{PN^2}$.

572. What regards the *Diſtinctneſs of Viſion* in compound Microſcopes, is derived from the Conſideration of the *Angles of Aberration*, both by *Colours* and the *Figures* of the Glaſſes. We have found (455) that the lateral Aberration DS of a Ray AQ parallel to the Axis occaſioned by its different Refrangibility, is to ds that of a Ray Aq proceeding from ſome Point q in the Axis, as the focal Diſtance of the Lens DF is to the Diſtance Bf. That is, SD : sd :: BF : Bf;

Fig. IX.

: Bf; and ſo $sd = \frac{SD \times Bf}{BF}$. But $SD = \frac{1}{55}$ AB (461) therefore $sd = \frac{\frac{1}{55} AB \times Bf}{BF}$, or sd is always as $\frac{AB \times Bf}{BF}$.

573. Therefore in the preſent Caſe of the Object Lens PD, the lateral Aberration of the Ray BD at N will be as $\frac{PD \times PN}{PO}$; and the Angle which this Aberration ſubtends at the Eye Glaſs E, will be larger in Proportion as the focal Diſtance thereof of NE is leſs. Therefore this Angle of Aberration by Colours will be as $\frac{PD \times PN}{PO \times NE}$.

574. The lateral *Aberration by the Figure* of the Object Lens PD is deducted in like Manner; for in this Caſe we had PQ as $\frac{NM^3}{BF^2}$ (476); that
Fig. IX. is, DS is as $\frac{AB^3}{BF^2}$; but in this Sort of Aberration, it was ſhewn, that SD : sd : : BF : Bf; and ſo SD is as $\frac{BF \times sd}{Bf}$; whence $\frac{AB^3}{BF^2} : \frac{BF \times sd}{Bf}$, therefore $AB^3 \times Bf = BF^2 \times sd$. Conſequently we have this Aberration $sd : \frac{AB^3}{BF^3} \times Bf$; and ſo the Angle which this Aberration ſubtends at the Eye-Glaſs E will always be as $\frac{PD^3 \times PN}{PO^2 \times EN}$.

575. As to the Aberrations by the ocular Lens, they are eaſily inveſtigated and compared with thoſe above, but they are ſo very ſmall as to be altogether inſignificant, and of Courſe it can be of no Uſe to waſte any Time about them.

CHAP.

CHAP. VIII.

The THEORY and CONSTRUCTION of *Compound* MICROSCOPES explained and exemplified.

576.

FROM the Principles we have premiſed, it will eaſily appear that a Microſcope may be conſtructed to magnify in any propoſed Degree with the ſame *Brightneſs and Diſtinctneſs of Viſion* as in a given *Standard Microſcope*, whoſe Conſtruction has been found by Experience to be the beſt of that Size. To this End the focal Diſtance of the ocular Lens in the Microſcope to be conſtructed muſt be given, and then the focal Diſtance of the Object Lens, its Aperture, and Poſition, will become known. *Note*, the Diſtinctneſs of Viſion is to be underſtood in regard of *one Sort of Aberration only*; for both Sorts cannot be the ſame as in the Standard Microſcope, and yet admit of a magnifying Power at Pleaſure, or, indeed, in any conſiderable Degree at all, as will be ſeen by and by.

577. Let the ſeveral Parts given in the *Standard* MICROSCOPE be repreſented by capital Letters, and by ſmall Letters in the ſmall One to be made as below.

Standard

Standard Microſcope.		*Small Microſcope.*
The Object Lens P	Ditto —	p
Its focal Diſtance PO = C	——	$po = c$
The ocular Diſtance E	——	e
Its focal Diſtance EN = D	——	$en = d$
Aperture of Ob. Lens PD = A	——	$pd = a$

And let BO : BP : : 1 : m, but $bo : bp$: : 1 : n. Alſo let the Optic Angle NEY, or apparent Magnitude in the Microſcope, be to the Angle BAX, or Angle ſubtended at the naked Eye, at the Diſtance AB, be to each other in the Ratio of M to q.

578. Then to conſtruct the ſmall Microſcope that ſhall magnify in a given Ratio, and in which the *Angle of Aberration by Colours* ſhall be the ſame as in the Standard one, you proceed in the following Method to determine the Object Lens p, the ocular Lens e being given. Since the Brightneſs of the Picture in each Microſcope muſt be the ſame, we ſhall have $\frac{PD \times NE}{PN} = \frac{pd \times ne}{pn}$ (571) that is $\frac{AD}{mC} = \frac{ad}{nc} = f$, (for greater Simplicity in the Operation).

579. Again, ſince the *Angle of Aberration by Colours* muſt be the ſame in both Microſcopes, we have $\frac{PD \times BP}{NE \times BO} = \frac{pd \times bp}{ne \times bo}$, (567, 573) that is, $\frac{mA}{D} = \frac{na}{d} = g$.

580. By theſe two Equations (of f and g) we get $\frac{ncf}{d} = a = \frac{dg}{n}$; and from hence we have $nn = \frac{dd}{c} \times gf = \frac{dd}{c} \times \frac{mm}{DD} C$; and by extracting the Root, we get $n = \frac{md}{D} \sqrt{\frac{C}{c}}$.

581. If this Value of n be ſubſtituted in the Equation of (580) we ſhall have $a = A\sqrt{\frac{c}{C}}$, or $\frac{a}{A} = \sqrt{\frac{c}{C}}$, whence $A : a :: \sqrt{C} : \sqrt{c}$; that is, in theſe Microſcopes, *the Diameters of the Apertures of the Object Lenſes are in the Sub-duplicate Ratio of their focal Diſtances.* This was the Caſe of Teleſcopes (517) and theſe Microſcopes become Teleſcopes by ſuppoſing BO and bo infinite, in which Caſe $m = n = 1$. Then alſo $\frac{A}{D} = \frac{a}{d}$, which gives $= \frac{d}{D} = \frac{\sqrt{c}}{\sqrt{C}}$, as in Article (518) was ſhewn.

582. In a given Diſtance of the Object AB, we have $q = \frac{ob \times ne}{op}$ (567), and becauſe $bo : bp :: 1 : n$, therefore $bp - bo\ (= op) : bo :: n - 1 : 1$; whence $\frac{ob}{op}\, ne = \frac{1}{n-1} \times d = \frac{d}{n-1} = q$; and thence $\frac{d+q}{q} = n = \frac{md}{D}\sqrt{\frac{C}{c}}$; and ſo $\frac{\overline{d+q}^2}{q^2} = \frac{mmdd}{DD} \times \frac{C}{c}$; and from thence we obtain $c = \frac{m^2 d^2}{D^2} \times \frac{q^2}{\overline{q+d}^2} \times C = po$, the *focal Diſtance* required.

583. *The Angles of Aberration by the Figure of the Object Lenſes in theſe Microſcopes, are reciprocally as their focal Diſtances; and therefore reciprocally in a duplicate Ratio of their Apertures* (517). For the Angle of Aberration by the Figure is as $\frac{PD^3}{PO^3} \times \frac{PN}{NE} = \frac{mA^3}{C^3 D}$ (574), and therefore in the new Microſcope, it is as $\frac{na^3}{c^3 d}$; and be-

cauſe $\frac{m\,A}{D}=\frac{n\,a}{d}$ (579), therefore theſe Angles will be as $\frac{A^2}{C^2}$ to $\frac{a^2}{c^2}$, that is, as $\frac{1}{C}$ to $\frac{1}{c}$; becauſe $A^2 \cdot a^2 :: C : c$, by (581).

584. The Value of $c = po$ (518) may be thus expreſſed $\frac{m^2 C}{D} \times \frac{d^2}{1 + \frac{2d}{q} + \frac{d^2}{q^2}}$, and it will from thence appear, that in Proportion as q is leſs, the Diviſor of this Fraction will be greater; and conſequently c, or focal Diſtance of the Object-Lens, will be leſs. Therefore by this Conſtruction we might magnify to any Degree, were it not for the *Aberration by the Figure.*

585. To give HUGENIUS's own Example of this Conſtruction; in his *Standard Microſcope* we have $NE = D = 2$; $PD = A = \frac{1}{20}$; $PO = C = \frac{1}{10}$; $PB = \frac{7}{9}$; whence $m = 10$, and $Q = \frac{D}{m-1} = \frac{2}{9}$ (582). So that the magnifying Power of the *Standard Microſcope* (ſuppoſing $AB = 8$ Inches) was as 8, to $\frac{2}{9}$, that is, as 36 to 1. Therefore that the new Microſcope may magnify twice as much, we muſt put $q = \frac{1}{9}$; then if $d = D = 2$, we ſhall have $po = c = \frac{70}{361}$; $n = 19$; $pb = \frac{35}{171}$; and $pd = a = \frac{1}{38}$. If we make $d = 1$, then we ſhall find $po = \frac{7}{40}$; but before it was $\frac{7}{36}$ nearly; therefore the *Alteration of the Eye Glaſs makes little Difference in the magnifying Power.*

586. In order to remove the Obſtacle ariſing from the *Aberration by the Figure,* we muſt make that Aberration the ſame in both Microſcopes; and then we have (583) $\frac{m\,A^3}{C^2\,D}=\frac{n\,a^3}{c^2\,d}=b$ (to ſhorten

the Work.) And the Brightneſs being the ſame in both, gives $\frac{AD}{mC} = \frac{ad}{nc} = f$, as before (578). Hence we have $a = \frac{ncf}{d}$, and $a^3 = \frac{n^3c^3f^3}{d^3} = \frac{ccdb}{n}$; this gives $n^4 = \frac{d^4}{c} \times \frac{b}{f^3} = \frac{d^4}{c} \times \frac{m^4}{D^4} \times C$: and therefore $n = \frac{md}{D}\sqrt[4]{\frac{C}{c}}$. Hence we have $a = \left(\frac{ncf}{d} =\right) A\sqrt[4]{\frac{c^3}{C^3}}$. Alſo becauſe $q = \frac{d}{n-1}$ (582) we have $\frac{d+q}{q} (= n) = \frac{md}{D} \times \sqrt[4]{\frac{C}{c}}$; therefore $\frac{\overline{d+q}|^4}{q^4} = \frac{m^4d^4}{D^4} \times \frac{C}{c}$; and conſequently $c = \frac{m^4d^4}{D^4} \times \frac{q^4}{\overline{d+q}|^4} \times C$.

587. Hence, becauſe $\frac{a}{A} = \sqrt[4]{\frac{c^3}{C^3}}$, *the Diameters of the Apertures of the Object Lenſes are as the Biquadrate Roots of the Cubes of their focal Diſtances.*

588. With reſpect to the Angles of Aberration by Colours in theſe Microſcopes, ſince they are as $\frac{mA}{D}$ to $\frac{na}{d}$ (579), and ſince in the preſent Conſtruction we have $\frac{mA^3}{DC^2} = \frac{na^3}{dc^2}$, and alſo $A : a :: \sqrt[4]{C^3} : \sqrt[4]{c^3}$, we ſhall, by Subſtitution, find $\frac{mA}{D} : \frac{na}{d} :: \sqrt{C} : \sqrt{c}$; That is, *the Angles of Aberration by Colours will be directly in a ſubduplicate Ratio of the focal Diſtances of the Object Lenſes.*

589. Therefore ſince one of theſe Aberrations is the ſame in each Microſcope, and the other continually decreaſes in the ſmall one, 'tis evident there can be nothing to obſtruct the Progreſs of magnifying by Microſcopes of this Conſtruction, but the Minuteneſs of the Object-Lens. For the focal Diſtance $po = c$ is in this as $\frac{d^4 q^4}{\overline{q+d}|^4}$, but in the foregoing Conſtruction of the Microſcope po was as $\frac{d^2 q^2}{\overline{q+d}|^2}$, (582), but the former of theſe Quantities is in a duplicate Ratio of the latter; and therefore the firſt Conſtruction is preferable to the ſecond, 'till the Aberration by the Figure comes to be ſenſible.

590. In the firſt Conſtruction we had $po = \frac{7}{38} = c$ nearly (585); but in this it will be $\frac{7000}{\overline{19}|^4}$, or $\frac{1}{19}$ nearly, putting $d = D = 2$, $q = \frac{1}{9}$, &c. but if $d = 1$, then in the preſent Microſcope we have $po = \frac{7}{160}$, in the former it was $\frac{7}{40}$, *viz. four times as long* for the ſame Value of q, or Power of magnifying.

591. If it were required to conſtruct a Microſcope in which the Aberrations of both Kinds ſhall be the ſame as in the Standard One, the ſame Object-Lens muſt be retained; for ſince in this Caſe $\frac{m\,A^3}{C^2 D} = \frac{n\,a^3}{c^2\,d}$ (586) if for n and a we ſubſtitute their Values found when $\frac{m\,A}{D} = \frac{n\,a}{d}$ (578) we ſhall have $c = C$, or the Object Lens the ſame in both.

592. Conſequently, ſince the focal Diſtance $po = c$ is a conſtant Quantity, in the Expreſ-

ſion for it (582) $c = \frac{md}{D} \times \frac{q^2}{\overline{d+q}^{\,2}} \times C$, we muſt make $\frac{md}{D} \times \frac{q^2}{\overline{d+q}^{\,2}} = 1$, by which we get $d = \frac{1}{\frac{m}{D} - \frac{1}{q}}$, and from thence it appears that by diminiſhing q, or encreaſing the magnifying Power, the focal Diſtance of the ocular Lens $d = ne$ will encreaſe at the ſame Time.

593. But in this Caſe if the ſaid focal Diſtance d were encreaſed to Infinity, or $\frac{m}{D} - \frac{1}{q} = o$, we ſhould then have $q = \frac{D}{m}$, and in the Standard Microſcope we have $Q = \frac{D}{m-1}$ (582). Wherefore $Q : q :: m : m - 1$, that is, *the magnifying Power by the new Microſcope is to that of the* Standed One as m to $m - 1$. Hence nothing can be done to any Purpoſe in this Mode of Conſtruction.

594. To give an Idea of the Size of theſe New Microſcopes; we had $d = 1$, and $q = \frac{1}{9}$ (585), then $\frac{d+q}{q} = n = 10$; alſo $po = \frac{7}{40}$. Hence $n - 1 : 1 :: op : ob$; that is, $9 : 1 :: \frac{7}{40} : \frac{7}{360} = ob$ (582), therefore $ap + bo = bp = \frac{7}{36}$. And $bo : po :: bp : pn$ (567), that is, $1 : 9 :: \frac{7}{36} : \frac{7}{4} = 1{,}75 = pn$. Hence the Length of the new Microſcope is $ve + in + pn + pb = 1 + 1 + 1{,}75 + 0{,}2 = 4$ Inches nearly, which is but a *third Part of the Length of* HUGENIUS's *Standard Microſcope, yet magnifies twice as much with equal Brightneſs and Diſtinctneſs.*

CHAP.

CHAP. XI.

A CALCULATION of the QUANTITIES of both KINDS of ABERRATION in the STANDARD MICROSCOPE.

595.

BECAUSE the Diſtinctneſs of Viſion is the principal Thing to be regarded in Microſcopes, where the ſmalleſt Objects are to be ſo extremely magnified, we ought to be ſatisfied to what Length we may proceed before the joint Effect of both the Aberrations begin to diſturb it, and thereby render Viſion imperfect. In order to this it will be neceſſary to calculate the Quantity of each Kind in that *Standard* MICROSCOPE of *Hugenius* which we have propoſed as a *Module* for all others.

596. And firſt for the *Aberrations by Colours* let P be the Object Lens (conſidered without Thickneſs) whoſe focal Diſtance is PO = $\frac{7}{10}$ of an Inch; and Semi-Aperture PD = $\frac{1}{20}$. Join DO. Then from what we have formerly delivered (461) it is evident, that a Ray parallel to the Axis, and incident upon the Lens in the Point D will be diſſipated into the Angle

O D *b* $= \frac{1}{55}$ DOP. Let O *b* be perpendicular to the Axis which is interſected by D *b* in *d*; therefore we ſhall have O *b* $= \frac{1}{55}$ PD $= \frac{1}{55} \times \frac{1}{20} = \frac{1}{1100}$; and hence OD (or OP $= \frac{7}{10}$) is to O *b* as 770 to 1.

597. It was alſo ſhewn that the incident Ray ND will be diſſipated through an Angle B D *q* = O D *b*, the Point B being the conjugate Focus to the Point N. And, becauſe alternately, the incident Ray BD is diſſipated thro' the Angle NDK = B D *q* (459) = O D *b*; it will alſo be DN (or PN) to NK as DO (or PO) to O *b*; that is, PO : O *b* : : 770 : 1.

598. But in this Standard Microſcope, EN : NP : : 2 : 7; and we had above NK : NP : : 1 : 770. Conſequently EN or MK is to NK as 770 × 2 to 7, or as 220 to 1. Therefore ſay, as 220 is to 1, ſo Radius to the Tangent of 15′ 38″.

599. To the Angle now found we muſt add a ſmall Angle N I *k* into which a Ray LI parallel to the Axis is diſſipated by the Eye-Glaſs EM. The Quantity of this Angle is thus found, we have PN : NE : : PD : EI, that is, 7 : 2 : : $\frac{1}{20} : \frac{1}{70}$ = EI; but N *k* $= \frac{1}{55}$ EI $= \frac{1}{3850}$; therefore ſay, As NE = 2 is to N *k* ſo is Radius to the Tangent 30″ = N I *k*, which added to 15′ 38″ make 16′ 8″ for the whole Aberration by Colours.

600. And ſince in all Microſcopes of the firſt Conſtruction (578) this Quantity of Aberration by Colours is the ſame, it remains to find how far we can proceed before we are ſtopped by the Aberration cauſed by the Figure of the Object-Lens PD, for that only can now diſturb Viſion, ſince the above Aberration by Colours admits of

its being very diſtinct, even though it be joined with the Aberration by the Figure.

601. To find therefore what the Quantity of this Aberration by the Figure is in the *Standard Microſcope*, where it does no Harm, we proceed thus: Let the Thickneſs of the Lens be TP, which muſt in this Matter be conſidered becauſe (as we ſhall hereafter ſhew) if the ſaid Object-Lens PD be a *double and equally concave One*, the Aberration by the Figure will be $\frac{5}{3}$ of TP, but if it be a *Plano Convex Lens*, and the Convex Side turned towards the Object BX, the Aberration will be $\frac{9}{2}$ of TP (467), but if the plain Side be towards the Object, then the Aberration will be only $\frac{7}{6}$ of TP, which is the leaſt it can be, and this is therefore the beſt Poſition of the Lens PD, as well as the beſt form of it.

602. It will alſo hereafter be ſhewn, that PO : PD : : PD : TP; that is, $\frac{7}{10} : \frac{1}{20} : : \frac{1}{20} : \frac{1}{280} =$ TP. But $\frac{7}{6}$ of $\frac{1}{280}$ is the Aberration O *d* of a Ray parallel to the Axis and incident on the Point D, that is, O*d* $= \frac{1}{240}$. Then *d* P or OP : PD : : O *d* : O *b*, that is, $\frac{7}{10} : \frac{1}{20} : : \frac{1}{240} : \frac{1}{3360}$ = O *b*. Then ſince the Ray ND makes the Angle of Aberration D B *q* = O D *b*; and alternately, the Ray BD will therefore make the Angle of Aberration NDK = B D*q*, or O D *b*, we have DO or PO : O *b* : : DN (= PN) : NK, that is, $\frac{7}{10} : \frac{1}{3360}$: : 2352 : 1 : : PN : NK; but EN : NP : : 2 : 7; by theſe two Analogies we have EN to NK, or MK to NK, as 672 to 1, that is, as Radius to the Tangent of 5′ 8″, the Quantity of the Angle NMK ſought.

603. But HUGENIUS found that his Microſcope would bear a much larger Angle of Aberration than this without any Detriment to diſtinct

Viſion, for by turning the convex Side of the *Plano-Convex Lens* downwards, the Angle of Aberration became $\frac{9}{2}$ of T P by a Ray (parallel to the Axis) falling upon the Point D. And yet he could ſcarce perceive there was any Diminution of the Diſtinctneſs of the View; tho' $\frac{9}{2}$ is near 4 times as much as $\frac{7}{6}$ of TP, or the Angle of Aberration by the Figure was about 20′ in this Caſe.

604. Therefore the *Hugenian* or *Standard Microſcope* bears an Angle of *Aberration by Colours* of 16′, and one *by the Figure* of 20′ at the ſame Time, with ſufficient Diſtinctneſs of Viſion; and therefore the New Microſcopes of the firſt Conſtruction (578) are to be uſed till the Aberration by the Figure of the Object Lens exceeds 20′.

605. After the ſame Manner you find the Quantity of the Angle of Aberration FDQ in the Teleſcope, for let the focal Diſtance of the Object-Glaſs be CF = 6 Feet, and Semi-aperture AC = $\frac{1}{2}$ of an Inch; then ſince QF = $\frac{1}{55}$ AC = $\frac{1}{110}$, and ſuppoſing the focal Diſtance of the Eye-Glaſs QD = $\frac{3}{2}$ of an Inch, we ſhall have QD : QF : : $\frac{3}{2}$: $\frac{1}{110}$: : 165 : 1. So is Radius to the Tangent of 21′, nearly. *Plate* VI. *Fig.* VII.

606. By Experience it is found that a ſmall Object Lens about $\frac{1}{10}$ of an Inch focal Diſtance may be uſed in a Compound Microſcope of the firſt Conſtruction. Whence putting $d = 1$, $m = 10$, $D = 2$, $C = \frac{7}{10}$ and $c\ \frac{1}{10}$, we find $n = 13{,}23$ (582) and therefore $q = \frac{1}{12.23}$, whence $Q : q : : \frac{2}{9} : \frac{1}{12.23}$: : 24,46 : 9; and therefore the magnifying Powers will be as 9 : 24,46 : : 36 : 96, that is, the ſmall Compound Microſcope magnifies an Object 96 times in Length and Breadth, and

ſo exceeds the Power of the Lens by itſelf, which is but 80 (562), beſides all other Advantages attending ſuch a Conſtruction. And becauſe q is as $\sqrt{c}$, 'tis evident, that an Object Lens $\frac{1}{10}$ of an Inch focal Diſtance will in ſuch a Microſcope give a magnifying Power of twice 96, or near 200 times in Length and Breadth, which is ſufficient for moſt Purpoſes.

CHAP. X.

The CONSTRUCTION *of a* Solid PERSPECTIVE *without either* LENS *or* SPECULUM.

607.

AS a PERSPECTIVE (whether *Teleſcope* or *Microſcope)* conſtructed *without either Lens or Speculum*, is an Idea not very familiar to us, we ſhall here endeavour to eludicate it, and ſhew by *Theory*, how a Solid of a diaphanous Subſtance, of a proper Length, and terminated at each End by ſpherical Surfaces, may, by duly proportioning the Radii of thoſe Surfaces become a *magnifying Perſpective univerſally* for every Sort of Object.

Plate VII. *Fig.* IX. 608. Let WXYZ be the Section of ſuch a Solid (of *Glaſs*, *Cryſtal*, *Water*, &c.) and let the End W Z be the Arch of a Circle deſcribed on the Center C with the Radius

CI; and the other End XY, a circular Arch deſcribed upon the Center N with the Radius NO. Alſo let the Axis of the Perſpective Solid be HFNP.

609. Then a Ray GD, parallel to the Axis, will be refracted to the Focus F, ſo that IF = 3 CI, if the Solid be of *Glaſs*; or if it be of *Water*, IF = 4 CI; for the Theorem $\frac{mdr}{md - nd - nr} = f$ (400), when d is infinite, gives $mr = f$; but r = CI, f = IF, and m = 3 in Glaſs, and = 4 in Water. Therefore, &c.

610. Hence F is the Focus of all Rays parallel to the Axis. Through the Center C draw any Line KCA, and it will be the Axis to all Rays HD parallel thereto, and by making EA = 3 EC, or CA = 2 CE, it will give the Focus A of thoſe oblique Rays.

611. Among the Rays laſt mentioned, that which paſſes through a Point H in the Axis of the Solid, making HI = 2 IC, will be refracted into DA parallel to the ſaid Axis HP. For in the above Theorem, if we put $d = 2r$, and $m = 3$, we have $\frac{6r}{2r - 2r} = f = \infty$; and conſequently the Point H is the Focus of all Rays AD within the Solid parallel to its Axis.

612. Therefore the Points AF and A will repreſent the two Points of a diſtant Object from whence the unrefracted Rays HI, KE are emitted, and ſo AF will be the Image of that Part of the Object contained between them.

613. The Angle ACF is the *Viſual Angle* under which the Object appears from the Cen-

ter C; this is equal to the Angle DHI or *Optic Angle* under which the Line AF is diſtinctly ſeen by the Eye at H. For ſince HD and AK are parallel Rays, they interſect the Axis in equal Angles, *viz.* ACF = DHI.

614. After the ſame Manner, if FO = 3 NO, then is F the Focus of Rays RF parallel to the Axis PO; and A the Focus of Rays parallel to the Axis ANQ; and AF the Image of a diſtant Object contained between the two Axis AQ and FP, and ſeen under the Angle ANF = MPO from the Center N.

615. From hence it is evident that ſince the Image AF is ſeen under the Angle DHI (= ACF) by Means of the Perſpective, it will appear ſo much larger than the Object itſelf which is ſeen under the Angle PNQ (= ANF) as FN is larger than CF. That is (putting NO = R = Radius of the Object End) the Ratio of the magnifying Power is that of 2 R to 2 *r*, or of R to *r*, or of NO to CI. The ſame as in a common Teleſcope with Lenſes.

Fig. X. 616. The ſame Demonſtration ſerves for the *Galilean Teleſcope*, by ſuppoſing the End WZ *concave* inſtead of *convex*, and of the ſame Degree or Radius CI. And thus a very curious manual Opera-Glaſs might be made of any tranſparent Subſtance.

Fig. XI. 617. If the Radius NO, inſtead of being very large, were ſuppoſed to be very ſmall, the Inſtrument then becomes a *Microſcope*, for if P be the Place of a ſmall Object but a little further diſtant from O than 2 NO, the Rays which proceed from a Point P will be refracted to a conjugate Focus F in the Axis, and other Rays emitted from any other Point Q will be refracted to the Point A in the Axis ANQ, and the Viſual Angle ſubtended by the Object PQ, and its Image AF is the ſame, or PNQ = ANF; but the other End of the Perſpective being the ſame as before, the Optic Angle DHI (= ACF) under which that Image is ſeen by the Eye at H will as much exceed the ſaid Viſual Angle ANF as FN exceeds FC, or in the Ratio of $\frac{FN}{FC}$. Again, the Object at P ſubtends an Angle PNQ as much greater than that it ſubtends at 8 Inches Diſtance, as 8 Inches exceeds the Diſtance NP or in the Ratio of $\frac{8}{NP}$. Therefore the Optic Angle DHI exceeds the Angle at 8 Inches Diſtance in the Compound Ratio of $\frac{FN}{FC} \times \frac{8}{NP}$; which is the ſame Ratio of magnifying Power as in a common Microſcope, with two Lenſes (76).

NEW

NEW
ELEMENTS *of* OPTICS.

PART VI.

CHAPTER I.

Containing the PRINCIPLES, and EXPERIMENTS of the PRISM, by which the RATIOS of the SINES of INCIDENCE and REFRACTION are determined in the *heterogeneous* RAYS of LIGHT.

618.

Plate VIII *Fig.* I.

LET ABC be the triangular Section of the *Priſm* moveable (in a Frame) about its Axis, and let HD repreſent a Beam of the Sun's Light falling upon the Side AB in the Point D; from thence it will be refracted to the other Side CB in the Part E; and from thence it will be refracted in a Beam a Beam of diverging coloured Light ERV to the oppoſite Side of the Room at ſome Height ZR above the Floor WZ.

619. As the Priſm ABC is moved about its Axis one Way and the other, the coloured *Spectrum* RV will be ſeen to move up and down upon the Wall or Screen, and there is *one Place* R to which it will deſcend, and no lower; for though the Priſm be kept turning the ſame Way, it will deſcend to that Point R, and there, after being *ſtationary*, it will begin to aſcend. Whence this Point R is called its *Station*.

620. In this Experiment, the ſolar Beam HD is ſhewn to conſiſt of *different Sorts of Rays*, as appears by their being *differently refrangible*, for ſome are refracted to R, and others to V; with all the intermediate Sorts between.

621. The Rays thus ſeparated by the Priſm, appear *differently coloured*; thoſe which are leaſt refracted, as ER, appear *red*; and ſuch as are moſt refracted, as EV, are tinged with a *Violet Colour*. Between theſe the Rays are of other Colours, *Orange*, *Yellow*, *Green*, *Blue*, and *Indigo*, in various Degrees of Strength.

622. When the coloured *Spectrum* RV comes to its Station at R; the Priſm is to be then ſtopped, and remain at reſt: in this Caſe, the Beam of Light DE within the Priſm, will be parallel to the Baſe thereof, or Side AC.

623. Hence it will follow, that the incident Beam HD, and the refracted one ER, are equally inclined to the Sides of the Priſm reſpectively; and of Courſe they make equal Angles with the Perpendiculars to thoſe Sides.

624. Let GT be drawn perpendicular to the Side AB in the Point D, and meeting the Line TB (which biſects the *refracting Angle* B of the Priſm) in the Point T. Then the Ray ED is to be conſidered as the *incident Ray*, and paſſing out of Glaſs into Air, will be refracted from the Perpendicular GD into the Direction DH, which is now to be conſidered as the *refracted Ray*.

625. Upon the Point D, as a Center, deſcribe the Semi-circle FGS; and continue out ED to O, and HD to M, which will interſect TB in I; then QDT (= ODG) is the *Angle of Incidence*; and IDT (= HDG) is the *Angle of Refraction*. From the Points O and N, let fall the Perpendiculars OK and NL, of which the former is the *Sine of the Angle of Incidence*; and the latter *the Sine of the Angle of Refraction*, ſuppoſe for the leaſt refrangible Rays ER.

626. The Proportion of theſe two Sines OK and NL is to be determined in different Sorts of Glaſs, which is eaſy to be done, thus; let the refracted Ray ER be produced, and it will cut the Line TB in the ſame Point I (as did the Ray HD) making the Angle IDE and IED equal, and both together are equal to the Angle MIR; but the Angle MIR + IRM = HMP = the *Sun's Altitude*, at the Time of the Experiment. Therefore HMP — IRM = MIR = 2 IDQ; or I DQ = ODN = $\frac{M - R}{2}$.

627. In order to determine this Quantity; the Sun's Altitude HMP is taken with a good *Quadrant*, and the Angle ERM is thus determined: let the Height EX and RZ be carefully meaſured, and alſo the Diſtance XZ on the

Floor; then make PR parallel to WZ interſecting EX in Y. Now the Part EY is known; alſo YR (= XZ) is known; therefore in the right-angled Triangle EYR, having two Sides given, the Angle R is known alſo; and conſequently the Angle NDG is then known, and its Sine NL.

628. Becauſe of the ſimilar Triangles TQD and DQB, the Angle QDT is equal to DBQ, which is known as being half the Angle ABC of the Priſm; and of Courſe is conſtantly the ſame. Hence then to find the Angles of Refraction NDG we have this

RULE.

To the Semi-Angle of the Priſm ($\frac{1}{2}$ B) *add half the Difference of the Sun's Altitude above the Horizon, and the Spectrum's Depreſſion below,* $\frac{M - R}{2}$; *and that Sum will give the Angle of Refraction* (NDG) *required.*

629. To give an Example of the Uſe of this Doctrine, the Experiment muſt be made with Priſms of *White Flint Glaſs*, and ſuch other Glaſs as we intend to combine with that in the Compound Object Glaſs of the Teleſcope; which is uſually that called *Crown Glaſs*, of an azure Colour.

630. Theſe two Sorts of Glaſs were cemented together and ground into the Form of a Priſm whoſe refracting Angle B was 50° 30′, the half whereof is 29° 15″ = QDT or ODG; the Height of the Sun when the Experiment was made was 47° 17″ = HMP. And having

meaſured

meaſured the Height $EX = 61\frac{1}{2}$ Inches, the Height $ZR = 47\frac{1}{2}$, and the Diſtance $XZ = 191$ Inches $= YR$, the Side EY is 14 Inches; and from thence the Angle ERY is found to be 4° 12′, in the *Priſm of White Flint Glaſs*. Then $47^\circ\, 17' - 4^\circ\, 12' = 43^\circ\, 5'$, the Half of which is $21^\circ\, 32'\frac{1}{2} = IDQ = NDO$. Then $29^\circ\, 15' + 21^\circ\, 32'\frac{1}{2} = NDG = 50^\circ\, 47'$, the Quantity of the Refraction of this Priſm, for the red Rays ER.

631. By the Table of Sines it appears that the Sine of Incidence OK is 4886, and the Sine of Refraction NL is 7748 of ſuch equal Parts as the Radius GD contains 10000. But $4886 : 7748 :: 1 : 1{,}5856 :: OK : NL$.

632. The ſame Experiment made with the Priſm of *Green* or *Crown Glaſs*, gives $RZ = 31\frac{1}{2}$ Inches; and from thence the Angle ERY is found to be 8° 55′, then $\frac{1}{2}M - \frac{1}{2}R = 19^\circ\, 11' = NDO$; therefore $NDG = 48^\circ\, 26'$; the Sine of which Angle is 7482. So that in this Priſm we have $OK : NL :: 4886 : 7482 :: 1 : 1{,}5312$.

633. In defining the Ratio of the Sines of Incidence and Refraction in the *red Rays* ER, there is no Difficulty or Uncertainty, as the Spectrum is on that Part pretty diſtinctly bounded; but in Regard to the *Violet Ray* EV, it is otherwiſe, they being of a darker Hue, grow fainter and fainter, and loſe themſelves by inſenſible Degrees: So that one can hardly ſay where the Spectrum on that Part terminates.

634. However (as we demonſtrated in a former Part of this Work, *Art.* 234.) that in Priſms of equal Aberrations, the refracting An-

gles muſt be in the *inverſe Ratio* of the Lengths of the Spectrums RV made by Priſms of *White Flint*, and *Crown Glaſs*, whoſe refracting Angles are the ſame, therefore it was only neceſſary to get a Priſm of *White Flint* with a refracting Angle, ſuch, that when applied to the Angle B of the Green Priſm, in an inverted Poſition, the Beam of Light which paſſed through them both ſhould be *equally refracted*, and of a round Figure on the Screen; alſo, of Courſe, that all Objects viewed through theſe two Priſms together ſhould appear quite free from Colours, except only what they naturally have.

635. In order to this it was neceſſary to make many Trials by grinding Priſms to different Angles till at laſt that was found which anſwered the Purpoſe perfectly well; and this Angle was within a few Minutes of 35 Degrees. Therefore the true Proportion of the coloured Spectrums RV in the equal Priſms of *Flint* and *Crown Glaſs*, were as $58^{\circ}\frac{1}{2}$ to 35, or as 5 to 3 nearly.

636. But as this Ratio differs conſiderably from the common Ratio of 3 to 2 (*Art.* 260.) I contrived to meaſure thoſe Spectrums themſelves in ſeveral Different Ways, as alſo thoſe of *circular Holes* of common Light, and I found the Ratio of 5 to 3 obtained in them all; and on the other Hand when thoſe Spectrums were made by the Green Priſm of $58^{\circ}\frac{1}{8}$ refracting Angle, and by the White Flint one of 35°, they are conſtantly of an equal Length, as nearly as could be eſtimated.

637. Since in different Sorts of Glaſs, *white*, *green*, &c. the ſpecific Refraction is undoubtedly variable; it will follow that the greater the

Difference of ſuch Refractions is, the better the two Sorts of Glaſs will be for the uſe of Teleſcopes; becauſe the Radius of each Lens will be greater for the *ſame focal Diſtance*, and conſequently the Error from the Figure will be leſs; but as this is a Point of great Conſequence in this *New Doctrine of Refractions*, I ſhall treat it more at large in the following Chapter.

CHAP. II.

EXPERIMENTS and CALCULATIONS for aſcertaining the *Ratio* of the fore-mentioned SINES in *White* FLINT and *Crown* GLASS.

638.

TO render this Matter moſt evident, the following Preparation is neceſſary. Let AEB be a Plano-concave Lens of *White Flint*, and DEC a Plano-convex of *Crown Glaſs*, both joined by their plain Surfaces CD; let G be the *Radiant Point*, from whence a Ray GD proceeds diverging to the Convex CD, and at D (ſuppoſing the Concave were away) the Ray would be refracted and diſſipated into the coloured Beam RDV, ſo that R would be the Focus of the *Red Rays* DR, and V the Focus of the Violet Rays DV. *Fig.* II.

639. Then 'tis plain, ſuch a Concave AB muſt be combined with the convex whoſe Focus of parallel Rays HD muſt be the ſame Point R and V for the Red and Violet when ſeparated, that is, ſuppoſe HD (a Ray parallel to the Axis ROG of the Lenſes) fall upon the Concave AB at the Point B, and be from thence refracted and diſſipated into the coloured Beam IDK, ſo that the red Rays DR, DK, the Violet Rays DV, DI, may be Parts of, or conſtitute the ſame Right Lines RK, and VI reſpectively; and ſo make the Angles RDV, IDK, alternate and equal to each other; alſo let O be the Focus of the parallel Rays HD incident upon the Convex CD, alone.

640. It is here ſuppoſed, that the Point of Interſection D of the Red and Violet Rays is in the contiguous plain Surfaces of the Lenſes; by this Means it will be unneceſſary to conſider the Thickneſs of the Compound Lens, being ſo exceeding ſmall as not ſenſibly to affect the focal Diſtances of Lenſes when they are large; and conſequently will allow of a Calculation more ſimple and perſpicuous.

641. Now putting

R = Radius of the Concave AEB
r = Ditto of the Convex CFD
F = the focal Diſtance FG.
D = the Diſtance ER
f = focal Diſt. FO of parallel Rays in CFD.
f = Ditto in the Concave
n = Sine of Incidence
m = Sine of Refraction

$$m - n = \left\{\begin{array}{l} a, \text{ for the Red Rays} \\ \mathrm{a}, \text{ for Violet Rays} \end{array}\right\} \text{ in Crown Glaſs}$$

$$m - n = \left\{\begin{array}{l} b, \text{ for Red Rays} \\ \mathrm{b}, \text{ for Violet Rays} \end{array}\right\} \text{ in White Flint.}$$

Then with reſpect to the Plano-convex Lens CD, we have this Theorem $\frac{nDr}{\overline{m-n} \times D - nr} = F$, or (putting $n = 1$,) $\frac{Dr}{aD - r} = F$. From whence we have this Equation $Dr = aDF - rF$, or $Dr + rF = aDF$; which gives this Analogy $D + F : D :: aF : r :: F : \frac{r}{a}$. That is, (in the Figure) RG : RE : : FG : FO.

642. But now ſince RE = RD, nearly; and FG = DG; therefore RG : DG : : RE : FO : : Angle KDG : Angle DRG (= HDR) : : $\frac{1}{HDR} : \frac{1}{KDG} :: \frac{RDV}{HDR} : \frac{IDK}{KDG}$; becauſe the Angles of Aberration RDV and IDK are equal by Suppoſition. Therefore it is evident, *That the Angle of Aberration* RDV *in reſpect of the total Refraction* HDR *in the Concave has the ſame Proportion to that of the Angle of Aberration* IDK *in regard of the total Refraction in the convex Lens, as the focal Diſtance of parallel Rays* RE *in the former has to the ſame focal Diſtance* FO *in the latter.*

643. Again, for a given focal Diſtance FG = F in the Convex CD, let the conjugate Focus's be R, V, as before, and the Diſtance RE = D, and VE = *d*, then we have $\frac{Dr}{aD - r} = \frac{dr}{\mathrm{a}d - r} = F$; this gives $\mathrm{a}dD - Dr = aDd - dr$, but $D = \frac{R}{b}$, and $d = \frac{R}{\mathrm{b}}$; conſequently, ſubſtituting theſe Values of D and *d*, we have $\mathrm{a}R - \mathrm{b}r = aR - br$, and ſo $\mathrm{a}R - aR = \mathrm{b}r - br$; and therefore this Analogy $R : r :: \mathrm{b} - b : \mathrm{a} - a$; That is, *The Radii of the Lenſes*

Lenſes have the ſame Proportion as the Differences of the Sines of Incidence and Refraction in the red and violet Rays in Priſms of equal Refracting Angles of White and Crown Glaſs.

644. Therefore alſo, *The Radii of the Lenſes have the ſame Proportion as the Angles of Diſſipation in Refractions by ſuch Priſms; and of Courſe the ſame Proportion as the Lengths of the coloured Spectrums produced thereby.*

645. To find how far this Theory agrees with Experiments, I meaſured the coloured Spectrums, as exactly as I could, from their ſenſible Limits, and found that of the White Flint Priſm 12½ Inches long, and 2½ broad; and that of the Priſm of Crown Glaſs about 8 Inches long, and 2 broad; and ſubducting the Breadths from the Lengths, the true Lengths of the Spectrums were 10 and 6 Inches.

646. Then by computing trigonometrically, as above, the Angles of the refracted violet Rays in each Priſm were found, and with thoſe of the red Rays, ſtand as below.

$$\text{Ang. of Incid. } 29^\circ\ 15' \left\{\begin{array}{lll} 50^\circ & 47' & \text{Red} \\ 52 & 17 & \text{Violet} \end{array}\right\} \begin{array}{l}\text{White}\\ \text{Priſm}\end{array}$$

$$\text{Difference} = 1\ \ 30\tfrac{1}{2} = 90',5$$

$$\text{Ang. of Incid. } 29^\circ\ 15' \left\{\begin{array}{lll} 48^\circ & 26' & \text{Red} \\ 49 & 18\tfrac{1}{2} & \text{Violet} \end{array}\right\} \text{Green}$$

$$\text{Difference} = 0\ \ 52',5$$

But 90,5 : 52,5 : : 5 : 3, very nearly.

647. This ſtill appears more exactly in a Compariſon of the Differences of the Sines of thoſe Angles, *viz.*

The

3:5::100:166

The Sine $52^\circ\ 17'\frac{1}{2} = 7910$ Violet
$50\ \ 47 = 7748$ Red

Difference 162

The Sine of $49^\circ\ 18'\frac{1}{2} = 7582$ Violet
$48\ \ 26 = 7482$ Red

Difference = 100

So that 162 : 100 : : 5 : 3 nearly, as before.

648. As in all theſe Caculations, the Sine of the Angle of Incidence is expreſſed by Unity, and ſo compared with the Sines of Refraction, we ſhall here ſet them down in that Manner as follows:

n m
~~n m~~

As 4886 : 7482 : : 1 : 1,5312, red Rays } Crown
4886 : 7582 : : 1 : 1,5518, Violet } Glaſs
Alſo 4886 : 7748 : : 1 : 1,5856, Red } White
4886 : 7910 : : 1 : 1,6189, Violet } Flint

$$\frac{5312}{15312} = \frac{332}{957}$$

$$\frac{5518}{15518} \quad \frac{2759}{7759}$$

Hence we have $\left\{\begin{array}{l} a = 0{,}5312. \\ \mathrm{a} = 0{,}5518 \end{array}\right\}$ Mean 0,5415

Difference is $\mathrm{a} - a =$ 206.

Again $\left\{\begin{array}{l} b = 0{,}5856 \\ \mathrm{b} = 0{,}6189 \end{array}\right\}$ 0,6022

Difference is $\mathrm{b} - b =$ 333

Therefore R : *r* : : 333 : 206 : : 5 : 3, very nearly.

3;5::206:343

103

2:3::206:309

618

649. In the ſame Manner it is ſhewn that the Radii of the Lenſes for a Compound Object Glaſs in Microſcopes muſt have the ſame Ratio of 5 to 3 to converge the Rays to a colourleſs Focus. For let ABDFC be ſuch a Glaſs; M the Place of the Object or Radiant Point, and G the Focus belonging thereto. If now (as before) we conſider the Ray GD as incident on the Plano-convex DFC alone, it will be thereby refracted and diſſipated into the beam RDV.

650. This very diſſipated Beam muſt alſo be that into which the Ray MD is refracted by the Plano-concave AEB; ſo that we ſhall have (putting D = EM) $\frac{\mathrm{DR}}{b\mathrm{D}+\mathrm{R}} = \mathrm{ER} = \frac{dr}{ad-r}$, and from thence $\frac{ad\mathrm{DR}-bd\mathrm{D}r}{\mathrm{D}+d} = r\mathrm{R}$ for *Red Rays*; and in the ſame Manner for Violet Rays we have $\frac{\mathrm{a}d\mathrm{DR}-\mathrm{b}d\mathrm{D}r}{\mathrm{D}+d} = r\mathrm{R}$; conſequently $\mathrm{aR}-\mathrm{b}r = a\mathrm{R}-br$, and ſo $\mathrm{aR}-a\mathrm{R} = \mathrm{b}r-br$; and therefore $\mathrm{R} : r :: \mathrm{b}-b : \mathrm{a}-a$, as before.

651. If the Point G be ſuppoſed at an infinite Diſtance, then will the Rays from M be refracted parallel to the Axis, as DL, and ſtill produce *uncoloured Viſion* of the Object M in the Focus of the Compound Glaſs, for the above Demonſtration holds good in every Value of d (= FG) finite or infinite. Such Compound Glaſſes therefore would do extremely well for *Eye Glaſſes* in *Teleſcopes* and *Microſcopes* of all Sorts were it not for one inſuperable Obſtacle, *viz.* The *Shortneſs of the Radii* R and r, which are neceſſary to produce the ſhort focal Diſtances of thoſe Glaſſes, and which therefore cannot ad-

1.8 1.2 8° .58 .585
3.5

mit of an Aperture for a ſufficient *Field of View.*

652. Therefore in all Caſes of a Compound Lens for producing Viſion without Colours, the Ratio of the Radii R, r, of the Concave and Convex Lens muſt be that of 5 to 3; and then the Ratio of their Focal Diſtances for parallel Rays will be that of 3 to 2 nearly; becauſe, $R = bF$, and $r = af$ (435), therefore $R : r :: 5 : 3 :: bF : af$; whence $3bF = 5af$, or $F : f :: 2,65 : 1,755 :: 5 : 3,32 :: 3 : 2$ very near.

~~a=.5415~~
~~b=.6022~~
~~3b=1.8066~~
~~5a=2.7075~~

653. Having thus determined the Ratio of the *Radii* of the two Lenſes which make the Compound *achromatic* One; we ſhall in the next Place proceed to conſider the other Sort of Aberration which reſults from the Figure or ſpherical Form of the Glaſſes, inveſtigate its Quantity in any given Lens, and then ſhew how it may be effectually deſtroyed in Compound Object Lenſes for Teleſcopes, and Microſcopes.

~~5 : 3 :: 3~~
Put R = bF = 5, r = af = 3
R : r :: 5 : 3
5 : 3 :: 5 : 3 3bF = 5af
3bF =
a = .5313 5a = 2.6565
b = .5857 3b = 1.7571

CHAP.

CHAP. III.

An INVESTIGATION *of* HUGENIUS's THEOREMS *for determining the total* ABERRATION *of a* RAY *occaſioned by the* ſpherical FIGURE *of* SPECULUMS *and* LENSES.

654.

Plate VIII *Fig.* III.

LET IOB be a refracting Surface, and QI a Ray incident thereon from the Radiant Point Q in the Axis QK. Suppoſe this Surface IB ſpherical, and S the Center of the Sphere; then if the Point I be ſuppoſed very near the Vertex O, the Ray QI will be refracted in the *Denſe Medium* to *q*; but if it be conſidered as the extreme Ray of a Cone diverging from Q to the Point I at a Diſtance from the Axis, it will be refracted ſo as to cut it ſooner in K, and K*q* is the Aberration of ſuch diverging Rays.

655. If the Ray QI were parallel to the Axis QK; this Aberration would be leſs, *viz.* *yt*, and nearer to the refracting Surface IB; for $\frac{rmd}{md-nd-nr} = Oq$, and $\frac{rmd}{md-nd} = \frac{mr}{m-n} = Ot$. And by (456) we have $yt : Kq :: Oy^2 : OK^2$.

656. Through the Center S let another Axis RBM be drawn parallel to the incident Ray QI, which will be cut by the refracted Ray in Z. In this Axis take $BM = Ot$; and then ZM will be the Aberration of the Ray QI in the Axis RM. From S let fall the Perpendicular SA on QI produced; alſo, from I draw IC perpendicular to the Axis RM. Then it is plain (468) that the Aberration $yt : ZM :: IX^2 : IC^2$ (or AS^2) $:: QI^2$ or $QO^2 : QS^2$. Put $QO = d$, $OS = r$, $OX = x$, $IX = y$, $ZM = z$; then $QS = d + r$; and we have $z = \frac{\overline{d+r}|^2}{d^2} \times ty$.

657. If we put $m - n = g$, then when the Refraction is *out of Glaſs into Air* the Aberration $yt = \frac{m^2}{ng} x$, (467) and here, as the Refraction is out of Air into Glaſs, the Aberration $yt = \frac{n^2}{mg} x$. Therefore $z = \frac{\overline{d+r}|^2}{d^2} \times \frac{n^2}{mg} x$. And becauſe $Ot = \frac{m}{g} r = BM$, we have $BZ = \frac{m}{g} r - z$; and $SZ = \frac{m}{g} r - r - z = \frac{mr - gr}{g} - z = \frac{nr}{g} - z$.

658. By ſimilar Triangles QIK and SZK, we have $QK : SK :: QI : SZ$; and it is disjointly $QK : QS :: QI : QI - SZ$, that is (putting $QI = b$) $QK : d + r :: b : b + z - \frac{nr}{g}$, Or $QR = \frac{\overline{d+r} \times b}{b + z - \frac{nr}{g}}$.

659. Therefore when the Point I coincides with O, and K with q, we ſhall have $Qq =$

$d + r$

$\frac{\overline{d+r} \times d}{d - \frac{nr}{g}}$, becauſe, in this Caſe, the Quantity ZM vaniſhes. Conſequently the Aberration Kq $= \mathrm{Q}q - \mathrm{QK} = \frac{\overline{d+r} \times d}{d - \frac{nr}{g}} - \frac{\overline{d+r} \times h}{h - \frac{nr}{g} + z}$.

660. For Perſpicuity ſake, let $d + r = a$, $\frac{nr}{g} = b$; then K$q = \frac{ad}{d-b} - \frac{ah}{h-b+z} = a \times \frac{dz + \overline{h-d} \times b}{\overline{d-b} \times \overline{h-b+z}}$.

661. Upon the Point Q, with the Radius QI, deſcribe the Arch IF; and put OF $= e = h - d$, and $h = d + e$; then for $d - h$, and h, in the laſt Expreſſion for Kq, put their Values e and $d + e$, and we have K$q = a \times \frac{dz + be}{\overline{d-b} \times \overline{d-b+e+z}}$ and becauſe $\overline{d-b} \times \overline{e+z}$ is exceeding ſmall in reſpect of $\overline{d-b}^2$, it may be neglected without any ſenſible Error, therefore the Aberration Kq $= \frac{a}{\overline{d-b}^2} \times \overline{dz + be}$.

662. But ſince, from the Nature of the Circle, we have XO : XF : : QF : SO; and conjointly, OF : XO :: QS : QF, or QO; therefore $e = \mathrm{OF} = \frac{\overline{d+r} \times x}{d}$; if now for e and Z we ſubſtitute their Values in the laſt Expreſſion, we ſhall have K$q = \frac{\overline{d+r}|^2}{\overline{d-b}|^2} \times \frac{n^2d + n^2r + mnr}{dmg}$.

663. By the Circle we alſo have XO : XI : : XI : 2OS nearly; XO $= x = \frac{y^2}{2r}$; Therefore

the Aberration $Kq = \frac{\overline{d+r}^2}{\overline{d-b}^2} \times \frac{n^2d+n^2r+mnr}{2rdmg} y^2$. And ſuch are the Expreſſions of the Aberration Kq after the firſt Refraction into a deſer Medium.

664. If the Ray QI be reflected from the *convex* Surface IB, ſtill the Proceſs is the the ſame; but, as in this Caſe, *n* will be *negative*, and equal to *m*; the Expreſſion with theſe Alterations will become $Kq = \frac{\overline{d+r}^2}{\overline{d+\frac{1}{2}r}^2} \times \frac{1}{2}x$. Or elſe $\frac{\overline{d+r}^2}{\overline{d+\frac{1}{2}r}^2} \times \frac{1}{4}\frac{y^2}{r}$.

665. But the Ray QI, reflected from a *concave* Surface, will have an Aberration $Kq = \frac{\overline{d-r}^2}{\overline{d-\frac{1}{2}r}^2} \times \frac{1}{2}x$. Or $Kq = \frac{\overline{d-r}^2}{\overline{d-\frac{1}{2}r}^2} \times \frac{1}{4}\frac{y^2}{r}$.

666. And for parallel Rays, where *d* is infinite, we have $Kq = \frac{1}{2}x$, or $\frac{y^2}{4r}$, both in the *convex* and *concave Speculum*.

667. Let the Ray QI, in its Paſſage to K,
Fig. IV. meet with the ſecond Surface ED of a Lens in
the Point E, and ſuppoſe it refracted by the Air to the Focus f in the Axis QK; and let R be the Center of that ſpherical Surface, and RE = R, the Radius.

668. In this Caſe IE is a Ray converging to the Point K at the Diſtance DK = *D*; and let D*q* = D; alſo let 𝒬 be the conjugate Focus to *q*, and *k* the conjugate Focus to K, or that to which the Ray IE would be refracted if the Point E were extremely near the Vertex D of the Lens;

then it is plain, the whole Aberration of the Ray QI will be $\mathcal{Q}$f, conſiſting of the two Parts $\mathcal{Q}k$, and k f; each of which muſt be ſeparately found.

669. From the given Diſtances D and *D*, and the Radii of the Lens r and R, we find the focal Diſtances $D\mathcal{Q} = \frac{RnD}{gD + Rm}$ (400), and $Dk = \frac{RnD}{gD + Rm}$; putting n for m, becauſe the Refraction is now out of a rare into a denſer Medium; and alſo $n - m = g$, which is in this Caſe Negative.

670. Then $D\mathcal{Q} - Dk = \mathcal{Q}k = \frac{RnD}{gD + Rm} - \frac{RnD}{gD + Rm}$; which Quantity, (as $gD - gD$ is exceeding ſmall) will be duly expreſſed by $\frac{mnR^2}{\overline{gD + mR}|^2} \times Kq$.

671. Since k is the conjugate Focus to K, the Aberration of the Ray IK will be $\frac{\overline{D + R}|^2}{\overline{D + b}|^2} \times \frac{m^2 D + m^2 R + nmR}{2gRnD} \times y^2 = k$ f, by what is before demonſtrated (663).

672. Having thus obtained the Values of the ſeparate Aberrations Kq, $\mathcal{Q}k$, and k f, we can find the ſame in Numbers from a general Theorem derived from the above ſymbolical Expreſſions; for ſuppoſing the glaſs Lens ſuch that it is preciſely $m : n :: 3 : 2$, then will $m - n = g = 1$, and $b = \frac{nr}{g} = 2r$; therefore the firſt Aberration $Kq = \frac{\overline{d + r}|^2}{\overline{d - 2r}|^2} \times \frac{4d + 10r}{6rd} \times y^2$.

673.

673. Again, the Aberration $\mathcal{Q}k = \frac{6R^2}{\overline{D+3R}^2} \times Kq$ (670) $= \frac{\overline{d\times r}^2}{\overline{d-2r}^2} \times \frac{4d+10r}{6dr} \times \frac{6R^2}{\overline{D+3R}^2} \times y^2$. It has been ſhewn (400) that $\frac{3dr}{d-2r} = Oq = Dq = D$, whence (putting $v = d - 2r$) we ſhall have $\mathcal{Q}k = \frac{\overline{d+r}^2 \times \overline{2d+5r} \times 6R^2}{\overline{dr+Rv}^2 \times 27dr} \times y^2$.

674. Laſtly; ſince D is very near equal to D, we ſhall find no ſenſible Error in taking $k\mathrm{f} = \frac{\overline{D+R}^2}{\overline{D+3R}^2} \times \frac{9D+15R}{4DR} \times y^2 = \frac{\overline{3dr+Rv}^2}{\overline{dr+Rv}^2} \times \frac{\overline{9dr+5Rv}}{36dRr} \times y^2$; for here $b = 3R$.

675. Becauſe in the Expreſſion above for $\mathcal{Q}k$, we have $\frac{6R^2}{27dr} = \frac{8R^3}{36drR}$, therefore the latter ſubſtituted for the former will give $\frac{\overline{d+r}^2 \times \overline{2d+5r} \times 8R^3}{\overline{dr+Rv}^2 \times 36drR} \times y^2$; which is now of the ſame Denomination with the Expreſſion for the Error $k\mathrm{f}$; and the Sum of both (when the Value of v is reſtored, and the whole divided by $3d$,) will be

$$\mathrm{Qf} = \frac{\left.\begin{matrix}27r^3\\33r^2R\\13r^2R^2\\7R^3\end{matrix}\right\}\times dd \quad \left.\begin{matrix}-66r^3R\\-52r^2R^2\\+14rR^3\end{matrix}\right\}\times d \quad \begin{matrix}+52r^3R^2\\+52r^2R^3\end{matrix}}{12rRd^2 \times \overline{r+R}^2 - 48r^2R^2d \times \overline{r+R} + 48r^3R^3} \times y^2.$$

676. But this Value of the total Aberration Qf may be expreſſed more conciſely if we take in the Thickneſs of the Lens inſtead of it's Semidiameter y. For ſuppoſing the extreme Ray were refracted from the very Edge of the Lens,

then the two Lines IX and EG would coincide in one, and we ſhould have $OX : IX = y :: y : 2r$ (663) in the ſame Manner $XD : y :: y : 2R$; therefore $OX + XD = OD = X = \frac{y^2}{2r} + \frac{y^2}{2R} = \frac{2R + 2r}{4rR} \times y^2$, and therefore $y^2 = \frac{2rR}{R + r} \times x$, which being ſubſtituted for y^2 in the above, and due Reduction made, we ſhall have

$$Qf = \frac{\left.\begin{matrix} 27r^2 \\ 6rR \\ 7R^2 \end{matrix}\right\} dd \quad \left.\begin{matrix} -66r^2R \\ \times\ 14rR^2 \end{matrix}\right\} d \quad + 52r^2R^2}{6dd \times \overline{r + R}|^2 - 24drR \times \overline{r \times R} + 24r^2R^2} \times x.$$

677. When the Rays are parallel to the Axis of the Lens, then, ſince d is infinite, thoſe Terms which contain dd will be infinitely greater than all the reſt, which may on that Account be neglected, and the Expreſſions of the Aberration will be abbreviated to

$$Qf = \frac{27r^2 + 6rR + 7R^2}{12rR \times \overline{r + R}} \times y^2, \text{ or } = \frac{27r^2 + 6rR + 7R^2}{6 \times \overline{r + R}^2} \times x.$$

And theſe are the very Expreſſions for the Value of the Error occaſioned by the Figure of the Lens, which the famous *Hugenius* invented and publiſhed (but without full Demonſtration) about 100 Years ago.

CHAP. IV.

The THEORY of Aberrations reſulting from the various FORMS, POSITIONS and *Radii* of LENSES to be combined for a compound *Object* LENS.

678.

AS the Thickneſs of the Lens is more convenient than its Semidiameter to be uſed in the Expreſſion of this Aberration, it will be neceſſary to obſerve that this Thickneſs is equal to the *Sum or Difference of the verſed Sines of half the Arches which compoſe them.* This will be eaſy to apprehend from the Figures of the different Lenſes, which are as follow.

Fig. V. 679. Firſt; the *double and unequally convex Lens* ABCD, the *Sum* of whoſe verſed Sines BO, DO, make the Thickneſs of the Lens, or it is BO + DO = T. *Fig.* VIII. The ſame is to be obſerved with reſpect to the *double Concave*, in which it is, indeed, ſuppoſed that the Points B and D coincide, but in *real Glaſſes* this is impoſſible.

Fig. VI. *Fig.* IX. 680. Secondly; In the *Plano-convex* and *Plano-concave*, the Surface ADC becoming a Plane, the verſed Sine DO vaniſhes, and the Thickneſs is only the other verſed Sine BO.

681.

681. Thirdly; In the *Meniſcus* ABCD, the Difference of the verſed Sines BO and DO makes the Thickneſs T of the Lens, or BO — DO = T. This is alſo the Caſe in the *concavo-convex Lens*. Wherein the Points B and D are ſuppoſed alſo to coincide, or to be ſo inconſiderably diſtant as not to affect the Concluſions of the Theory. *Fig.* VII. *Fig.* X.

682. In the foregoing Theorems for the Aberration of Lenſes, it is ſuppoſed that $r =$ Radius of the firſt Surface ABC, or that which is expoſed to the Radiant, and $R =$ Radius of the ſecond or hinder Surface ADC, or *a* D *c*.

683. In a *double and equally convex* or *concave* Lens, we have $r = R$, and $\frac{27r^2 + 6rR + 7R^2}{6 \times \overline{r+R}|^2}$ $\times$ T becomes $\frac{5}{3}$ T = A, the Aberration in ſuch Lenſes.

684. If R be infinite, or the ſecond Surface a *Plane*, the Aberration will be $\frac{7}{6}$ of the Thickneſs of the Lens, or it will be $A = \frac{7}{6}$ T.

685. But when r is infinite, or the firſt Surface a Plane, the Aberration will be $\frac{9}{2}$ T = A, (as was formerly ſhewn by another Method (467.)

686. Hence it appears that the Aberration of a *Plano-convex* Lens, when the plain Side is turned to the Radiant, is near four times as great as it is when the convex Surface is expoſed to it; for $\frac{9}{2} : \frac{7}{6} :: \frac{54}{12} : \frac{14}{12} :: 54 : 14 :: 4 : 1$, nearly.

687. And the ſame holds good of a *Plano-concave*; and indeed it may be obſerved, once for all, that the Aberration, under all Circumſtances, is the ſame in Lenſes that are *Convex or Concave in an equal Degree*; and that therefore what is ſaid of one is true of the other alſo.

688. When R is negative, the Lens is a *Me-*

Fig. VII. *niſcus*, and then the Aberration will be $A = \frac{27r^2 - 6rR + 7R^2}{6 \times \overline{r - R}^2} \times T$.

689. Alſo, when *r* is negative, the Aberration becomes $A = \frac{27r^2 - 6rR + 7R^2}{6 \times \overline{R - r}^2} \times T$, for

Fig. X. the *Concavo-convex Lens.*

690. Therefore in the *Meniſcus* and *Concavo-convex Lens* the Aberration is the ſame if the Radii *r* and R be the ſame in each; becauſe $\overline{r - R}^2$ is the ſame as $\overline{R - r}^2$.

691. From the general Equation (677) it appears, there is no ſuch thing as a Lens of any Sort without any Aberration at all. For ſuch a Suppoſition the Theorem would be $A = \frac{27r^2 \pm 6rR + 7R^2}{6 + \overline{r \pm R}^2} = 0$, which gives $27r^2 \pm 6rR + 7R^2 = 0$; and therefore $\pm 6R + 7R^2 = -27$, (putting $r = 1$) and therefore $R = \pm \frac{3}{7} \pm \sqrt{\frac{9}{49} - \frac{27}{7}}$, which is plainly an impoſſible Quantity. Conſequently the Aberration A can in no Caſe be equal to Nothing.

692. Its Quantity therefore in ſome certain Lens will be a *Minimum*, which is eaſily diſcovered by throwing the general Equation (677) into Fluxions, and then making it Nothing, in the uſual Way; by this Means we have the Fluxion $-48\dot{R} - 40R\dot{R} + 8R^2\dot{R} = 0$; therefore $8R^2 - 40R = 48$, or $R^2 - 5R = 6$, which gives $R = 6$; the Radius *r* having been firſt put $= 1$.

693. Therefore the Lens whoſe Radius *r* and R are to each other as 1 to 6, has the leaſt Aberration of all others; and conſequently the *Convexes* and *Concaves of this Form*, for any given fo-

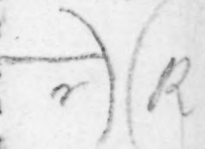
cal Diſtance, are the beſt of all others for optical Uſes of every Kind.

$\frac{15}{14} = 1.07143-$

$\frac{7}{6} = 1.16667-$

$x\ 0.09524$

694. What this leaſt Aberration is will appear if, in the general Theorem, we ſubſtitute 1 and 6 for r and R, which then becomes $\frac{15}{14}$ T = A, which, tho' the leaſt of all, is yet not much leſs than $\frac{7}{6}$ T in a Plano-convex, (684) for $\frac{15}{14} : \frac{7}{6} ::$ $15 \times 6 : 14 \times 7 :: 45 : 49 :: 15 : 16$, nearly.



695. This Aberration belongs to the Lens in its beſt Poſition, of Courſe; for if the Lens be inverted, or we put $r = 6$, and $R = 1$, then the Aberration will be $\frac{145}{42}$ T = A, which is more than three times as great as before. Hence it appears how ſtrict a Regard ought always to be had to the Poſition of Glaſſes in optical Inſtruments to render them moſt perfect in their Kind.

696. The Radii r and R of any double Convex Lens being given, the Aberration of that Lens is eaſily computed from the general Equation thus, ſuppoſe $r = 2$ and $R = 5$, then theſe Numbers being put for r and R, the Equation becomes $\frac{7}{6}$ T = A, the ſame as in a *Plano-convex* in its beſt Poſition, and of the ſame Thickneſs. (684).

697. The Aberration of a Lens may be alſo expreſſed by an Equation conſiſting of one Radius and the focal Diſtance F of the Lens. For becauſe $\frac{2rR}{r+R} = F$ (433) we have $R = \frac{rF}{2r - F}$; which being put for R in the original Equation, it becomes $\frac{27r^2 - 24rF + 7F^2}{6r^2} \times T = A$, which may alſo be thus expreſſed, *viz.* $\frac{9}{2}T - \frac{4F}{r}T + \frac{7F^2}{6r^2}T = A$.

698. Hence it is evident, that not only when

r is infinite, or the Lens a Plano-convex with its plain Side to the Radiant, the Aberration is $\frac{9}{2}$ T $=$ A, (as in 685) but alſo when r and R are ſo related that $\frac{4F}{r}T = \frac{7F^2}{6r^2}T$, or $4 = \frac{7}{6}\frac{F}{r}$, that is $24r = 7F$. Now in this Caſe we have $\frac{24r}{7} = F = \frac{2rR}{r+R}$, and therefore $24r = -10R$, and ſo $r : -R :: 10 : 24 :: 5 : 12$; or a Meniſcus whoſe Radii r and R are as 5 to 12, has an Aberration $A = \frac{9}{2}T$.

699. Again; if the Radius R of the ſecond Surface of a Lens, and the focal Diſtance F be given, the Aberration is thence found, becauſe $r = \frac{RF}{2R-F}$, which being ſubſtituted for r in the Equation (697) will give $\frac{7F^2}{6R^2}T - \frac{2F}{3R}T \times \frac{7}{6}T = A$. Hence again it appears, when R is infinite, the Aberration is $\frac{7}{6}$ T. And alſo when $\frac{7F^2}{6R^2} = \frac{2F}{3R}$, or $\frac{7F}{6R} = \frac{2}{3}$, which gives $21F = 12R$, and $\frac{12R}{21} = F = \frac{2rR}{r+R}$; whence we have $2R = 5r$, or $r : R :: 2 : 5$, the Ratio of the Radii of a double convex Lens whoſe Aberration is $\frac{7}{6}$ T, and therefore equally good with a Plano-convex of the ſame Thickneſs, or focal Diſtance.

700. In the ſame Manner, having one Radius and focal Diſtance of a *Meniſcus*, the Aberration is found; for it is then $A = \frac{7F^2 + 4FR + 7R^2}{6R^2} \times T$, when the convex Surface is turned to the Radiant, or $r = \frac{FR}{2R \mp F}$. Where it is eaſy to

ſee that the leſs R is, the greater will be the Aberration A; but how great ſoever we take R, yet A will always exceed $\frac{7}{8}$ T.

701. If the concave Side of the *Meniſcus* be turned towards the Object, or it be $R = \frac{rF}{2r + F}$; then the Aberration will be $A = \frac{27r^2 + 24rF + 7F^2}{6r^2} \times T$. Here again it is evident A will be greater as r is leſs; but 'till r is infinite, A will always be greater than $\frac{27}{6}$, or $\frac{9}{2}$ of T.

702. In the *Concavo-convex Lens*, if the concave Side be placed outwards, or $r = \frac{RF}{2R + F}$, then we have $A = \frac{7F^2 + 4FR + 7R^2}{6R^2} \times T$. Where it appears that the leſs we take R, the greater A will be; and that it will always be greater than $\frac{7}{6}$ T.

703. But if the *convex Side* be outwards, or $R = \frac{rF}{2r + F}$; then we have $\frac{27r^2 + 24rF + 7F^2}{6rr} \times T$. Whence it appears, the leſs r is, the greater A will be; and that it will ever exceed $\frac{9}{2}$ T.

704. But it will be proper to expreſs theſe two Caſes of the Poſition of a Convexo-concave in Terms of the Radii only, thus;

Caſe I. Concave Side outwards, $A = \frac{27r^2 - 6rR + 7R^2}{6 \times \overline{R - r}^2} \times T$.

Caſe II. Convex Side outwards, $A = \frac{27R^2 - 6Rr + 7r^2}{6 \times \overline{R - r}^2} \times T$.

Becauſe it is evident by Inſpection, that, ſince $27R^2 + 7r^2$ is greater than $27r^2 + 7R^2$, the Aberration is much greater in the ſecond Caſe than in the firſt; and that therefore the beſt Poſition of ſuch a Lens is *when the concave Side is towards the Radiant.*

705. In a Lens of any Form, the *Thickneſs, Breadth,* and *focal Diſtauce* are in geometrical Proportion; for we have ſhewn (676) that $y^2 = \frac{2Rr}{r+R} \times T$; and ſince $F = \frac{2rR}{r+R}$, therefore $y^2 = FT$; conſequently, $T : y :: y : F$. Hence if any two of theſe, in any Lenſes, are the ſame, the third will be the ſame alſo.

CHAP. V.

The foregoing THEORY reduced to PRACTICE in Calculations of the *Radii* and focal Diſtances of *Concave* and *Convex* LENSES for conſtituting the *Compound Achromatic* LENS.

706.

WE have now paſſed through all thoſe Parts of the Theory that are neceſſary for aſcertaining the true Forms and Poſitions of the Lenſes that are to be combined for conſtituting a compound Object Glaſs that ſhall be truly *Achromatic*, and at the ſame time free from the Frrors ariſing from the Aberration of Rays by a ſpherical Figure.

707. We muſt therefore now reduce thoſe Principles to Practice, and calculate the proportional and real Value of the Radii in thoſe

As $3 : 2 :: 1.162 : .775 = \frac{R}{2a}$ = focus Convex

2324 $\times 1.08$

$a = .54$ 6200

$2a = 1.08$ 7750

$R = .83700$ Probatum

Lenſes reſpectively that ſhall be requiſite for a compound Object Glaſs of a given focal Length. And to make this Matter as plain as poſſible, let us take firſt a convex Lens whoſe *Radii* are equal, and then it's Aberration will be $\frac{5}{7}$ of its Thickneſs $\mathcal{T}$ (683); and its focal Diſtance F = $\frac{R}{2a}$ (435).

708. The Lens of ***White Flint*** to be combined with this green Convex muſt be of the *Concave* Kind, becauſe its Aberration muſt not only be equal to that of the Convex, but likewiſe contrary to it, that it may entirely deſtroy it. The focal Diſtance of the Concave, $\frac{rR}{b \times \overline{r \pm R}}$, muſt alſo be to that of the Convex $\frac{R}{2a}$ as 5 to 3, to prevent any colorific Refraction (649.)

709. The Form of this Concave, or the Proportion and Quality of its *Radii* r and R, muſt be inveſtigated from an Equation of its Aberration to that of the Convex, *viz.* $\frac{5}{3}\mathcal{T} = \frac{27r^2 + 6rR + 7R^2}{r^2 + 2rR + R^2} \times \frac{T}{6}$. Now by putting R = 1, and ſince T : $\mathcal{T}$ :: 2 : 3 (652), the Equation becomes $15 = \frac{27r^2 + 6r + 7}{r^2 + 2r + 1}$, which gives this Quadratic $r^2 - 2r = \frac{2}{3}$; then by compleating the Square, and extracting the Root, we find r = 2,3. And hence the Ratio of the Radii is R : r :: 1 : 2,3.

710. Hence, ſince the focal Diſtances of the two Lenſes are as $\frac{R}{2a} : \frac{rR}{b \times \overline{r + R}}$ :: 2 : 3, we have the Radius of the double Convex R = 0,836.

P There-

Fig 11.

r = 2.3
× R = 1
rR = 2.3

r = 2.3
R 1
3.3
× b = .6
1.98

1.98)2.80(1.1
1.98
820
1220
1188

r = 2.29
r² = 5.2441
−2r = 4.58
.6641

b = .6

r² = 5.29
−2r = 4.6
.69

therefore r is [illegible] 2.3

Therefore if R = 10, we have $r = 23$; and $R = 8{,}36$.

711. The Rule for finding the compound focal Diſtance of both theſe Lenſes combined is derived from the Theory of the Lenſes themſelves, or from the Rule in (103), *viz. Divide the Product of the focal Diſtances of the two ſingle Lenſes by their Difference, the Quotient will be the compound focal Diſtance of both together.* By this Operation we get $\frac{RrR}{2arR - bR \times \overline{r + R}} = F$ $= 2{,}33$ or $23{,}3$, for the compound focal Diſtance, of the double *achromatic Object-Glaſs*, accordingly as we put R = 1 or 10.

Conc. 10 }
23 }
2) 33
16,5
Conv 8,36 —
8.14

10 × 23 × 8.36
= 1922

712. If we had put the Radius of the firſt Surface, or $r = 1$, we ſhould have had $R^2 + 3R = \frac{3}{2} = 1{,}5$; and thence $R = 0{,}44$ for the Radius of the ſecond Surface; but $0{,}44 : 1 :: 1 : 2{,}3$, the ſame as before.

$\frac{1922}{8.14} = 23.6$

713. This Combination of Glaſſes is repreſented, *Fig.* XI. where AB is the *double Concave* of *white Flint*, whoſe focal Diſtance for red Rays is ER; DC is the *double Convex* of equal Radii whoſe focal Diſtance is FO; and the compound focal Diſtance of both together is FG, for the parallel Rays HG, RF, *&c.*

714. It may be in many Caſes very convenient to expreſs the compound focal Diſtance by Unity, *viz.* making $FG = 1 = \frac{rRR}{2arR - bR \times \overline{r + R}}$. In this Caſe the Radii muſt each be diminiſhed in the ſame Ratio, *viz.* that of 2,33 to 1; and they will then become reſpectively $r = 0{,}99$, $R = 0{,}43$, and $R = 0{,}36$. Or if $F = FG = 100$, then $r = 99$, $R = 43$, and $R = 36$.

715.

1:1.10163.

715. Though a Double and equally convex Lens of green Glaſs is, on ſome Accounts, the moſt eligible Form; yet, if any other Form be propoſed, you may in the ſame manner (as before) determine the white Lens to be combined with it, to *correct its Aberration*, and render it *Achromatic*.

716. Thus, for Example, if the green Lens be a *Plano convex* in its beſt Poſition, its Aberration will be $\frac{7}{6}\mathcal{T} = A$ (by 684), then we have $\frac{7}{6}\mathcal{T} = \frac{27r^2 + 6rR + 7R}{r^2 + 2rR + R^2} \times \frac{T}{6}$; then putting $R = 1$, (and $T : \mathcal{T} :: 2 : 3$) we have $\frac{21}{2} = \frac{27r^2 + 6r + 7}{r^2 + 2r + 1}$; this Equation will give $33r^2 - 30r = 7$, from whence we obtain the Ratio of the Radii of the Concave, *viz.* $R : r :: 1 : 1{,}1$, or as 10 to 11; and becauſe it is $\frac{R}{a} : \frac{rR}{b \times \overline{r + R}} :: 2 : 3$ we ſhall have $R = 0{,}308$, the Radius of the *Plano-convex* given. Theſe two Lenſes are ſhewn in *Fig.* XII.

1×95 = .95

.95
+1.00
1.95
×.6
1.170

1.17)950(.812
936
140
230

3:2::.812:.541
1624

$\frac{rr}{ar+ar} = .541$

717. For another Example, let the given Convex be that of the beſt Form, whoſe Aberration (694) is $\frac{15}{14}\mathcal{T} = \frac{27r^2 + 6rR + 7R^2}{r^2 + 2rR + R^2} \times \frac{T}{6}$; then by Reduction, we have $\frac{135}{14} = \frac{27r + 6r + 7}{r^2 + 2r + 1}$; which Equation will give $243r^2 - 186r = 37$, whoſe Root is $r = 0.95$. Therefore the Ratio of the Radii of the required Concave is $R : r :: 1 : 0{,}95$. Theſe two Lenſes appear in *Fig.* XIII.

718. By a like Procedure you will find what concave Lens may be combined with any other given Form of a *Concave or Meniſcus whatſoever*. But this will not be the Caſe with regard to *given*

ar + ar = 2a+r+r

2a = 1.08

Concaves, for we are under ſuch Reſtrictions in their Forms, that a Concave may be propoſed for which no Convex can be found of an *equal Aberration*; and to ſhew theſe Limitations would be no very difficult Operation; but as it can be of little Uſe, we here paſs it by.

719. However, as the Reader may have the Curioſity (if not Occaſion) to know what Form of Lens of *Crown Glaſs* muſt be combined with a given Lens of *White Flint*, the Caſe ſhall be illuſtrated by an Example or two. Let the given Lens be a *Plano-concave* in its beſt Poſition, then its Aberration will be $\frac{7}{6}$ T and conſequently $\frac{7}{6}\,\mathrm{T} = \frac{27\,r^2 + 6\,r\,R + 7\,R^2}{r^2 + 2\,r\,R + R^2} \times \frac{T}{6}$; therefore (making $R = 1$) we have $\frac{7}{6}\,\mathrm{T} \times \frac{6}{T} = \frac{14}{3} = \frac{27\,r^2 + 6\,r + 7}{r^2 + 2\,r + 1}$, this Equation gives $67\,r^2 = 10\,r - 7$; by reſolving which it will be found that the Root r is impoſſible, and of Courſe no *Plano-concave* can, in its beſt Poſition, be uſed in an achromatic Object Glaſs.

720. But that the Reader may judge what *Concaves* may or may not be uſed, it will appear from the Theory, that if the Aberration of the given Concave be $\frac{a}{b}$ T, then $\frac{a}{b}\,\mathrm{T} \times \frac{6}{T} = \frac{4a}{b} = 6{,}42857$ will be a *Minimum*; whence $a : b :: 6{,}43 : 4 :: 16 : 10$ nearly. Therefore when $\frac{4a}{b}$ is leſs than 6,43 the Problem will be impoſſible. Thus $\frac{7}{6} \times 4 = 4{,}66$; and therefore, in the foregoing Inſtance, a *convex Lens* is not poſſible. Much leſs, ſtill, can a *Concave* of the beſt Form, and in its beſt Poſition, be thus uſed; becauſe its Aber-

Perhaps it may be better to have 2 Concaves

Aberration $\frac{13}{24}$ T, gives $\frac{4a}{b} = 4,284$ only.

721. But either of theſe two Concaves may be uſed in their worſt Poſition, as having then an Aberration ſufficiently large (686) to admit of a Convex to be combined with it. And by having this Cue, the young Optician may eaſily run over all the poſſible Varieties of Combinations of two ſuch Lenſes for conſtituting a *perfect achromatic Object Glaſs for a refracting Teleſcope of any given Length.*

N. B. A due Regard muſt be had to the Diſtinction of the Roman and Italic Letters R, *R* and T, *T*, in the foregoing Theorems.

CHAP. VI.

The THEORY of the *Achromatic* and other *Six-Glaſs* TELESCOPES explained; with a New, but Genuine, CONSTRUCTION thereof.

722.

THE Structure of the *achromatic* TELESCOPE is the ſame in all reſpects (but that of the *Object-Glaſs)* with that of the common *Six-Glaſs Teleſcope*, as it is uſually called; but becauſe there is a Circumſtance in this Conſtruction that affects this *achromatic* Object-Glaſs in a Manner not commonly thought of, it will be neceſſary to point out every Affection of ſuch a Teleſcope by conſidering minutely the Theory thereof from the *Geometrical* PRINCIPLES of OPTICS.

723. The Conſtruction of the Teleſcope is repreſented in one View, with the Diſpoſition of the Glaſſes as they have hitherto been made, wherein ABCD is the Concave of *white Flint*, and AEB the Convex Lens of *Crown Glaſs*; which both together conſtitute the *Compound achromatic Object-Glaſs* of the Refractor. *Fig.* XII. *Plate* VII.

724. The focal Diſtance of this Object-Glaſs is at F, which is found by (711). And the Image of vaſtly diſtant Objects would be there formed and repreſented by IM, if the parallel Rays (*a*, *b*, *c*,) were to paſs through the compound Object-Glaſs only; and it is the Image IM alone which can properly and truly be eſteemed *Achromatic or Colourleſs*.

725. But in the common Conſtruction of this Teleſcope, there is no ſuch Image as IM, but another (*im*) formed in another Focus G, by the Intervention of a Glaſs at Q, which intercepting the Rays tending to the Focus F, converges them to the ſecondary Focus G.

726. Now though it were poſſible to get white and green Glaſs, ſuch as would make an Image IM, perfectly achromatic by the Rays converging all to one Point F, yet this cannot poſſibly be the Caſe with reſpect to the Image (*im*;) becauſe the Rays, after paſſing through Q, a ſingle Lens, will in ſome ſmall Degree be diſſipated and refracted to different Points in the Axis, and therefore cauſe that the Image (*im*) is not ſo pure, uncompounded, and colourleſs as the Image IF, or as it is commonly imagined to be.

727. Having determined the focal Diſtance EF of the compound Lens, the Point G of the ſecondary Focus, in the beſt Poſition of the Lens Q, will be eaſy to be aſſigned, as it is $\frac{1}{2}$ QF = QG

when the focal Diſtance of Q is = QF. But this cannot be the Caſe, as ſuch a Poſition of the Lens Q would diminiſh the magnifying Power of the Teleſcope *full one Half of the Whole*, as it is thus demonſtrated.

728. By the two Theorems in (101, 102) we ſhall have the equivalent Lens $x = \frac{Fy - Ff}{y}$, (ſuppoſing F = EF = focal Diſtance of the compound Object-Lens.) Then it will be $F . \frac{Fy - Ff}{y} :: 1 : \frac{y - f}{y} :: 1 : 1 - \frac{f}{y}$, which is therefore the Ratio of the magnifying Power without and with the Glaſs Q; and becauſe $f = GQ = \frac{1}{2} y = \frac{1}{2} QF$, this Ratio $1 : 1 - \frac{f}{y}$ becomes $1 : \frac{1}{2}$. *Q. E. D.*

729. As the Value of $f = QG$ is known from the given Values of F, *y*, and D; ſo the Ratio of the magnifying Power $1 : 1 - \frac{f}{y}$ will in all Poſitions of Q become known, as far as it is independent of the Eye-Glaſſes; for this Glaſs Q is not to be conſidered as an Eye-Glaſs, though it be included in the ſame Caſe or Drawer with them.

730. For the Office of an Eye-Glaſs, ſingly, is only to give a diſtinct View of the Image IM, or *i m*, in its Focus. For the ſame Eye-Glaſs applied in different Teleſcopes does no ways affect their magnifying Powers.

731. Let the focal Diſtance of the ſingle Eye-Glaſs be C; then $\frac{x}{C} = M$, the magnifying Power of the Teleſcope; therefore $x = CM = \frac{Fy}{F + y - D}$,

and when $D = 0$, we have $M = \frac{F}{C} \times \frac{y}{F+y}$ a *Minimum.* And $M = \frac{F}{C}$, when a *Maximum*, or when $D = F$. Conſequently $\frac{M}{M} = \frac{F+y}{y}$.

732. From hence it appears, that by giving the Glaſs Q a moveable Situation, at Pleaſure, through the whole Length of F, you may vary the magnifying Power of the Teleſcope $\frac{F+y}{y}$ times. Suppoſe $F = 30$, and $y = 5$, then will the greateſt exceed the leaſt Power 7 times. This Glaſs Q, in all the achromatic and other Six-Glaſs Teleſcopes hitherto made, is fixed, and therefore this Liberty of varying the magnifying Power is loſt, and many other Advantages depending upon it, as we ſhall ſee farther on.

733. In the *Six-Glaſs Teleſcopes* there are three Eye-Glaſſes employed to rectify the Image (*i m*) by making thereof a Secondary Image NO, which is viewed through the Eye-Glaſs at P, a *Plano-convex Lens.* In Proportion as the Image NO is greater or leſs than the Image (*i m*), ſo theſe three Glaſſes R, S, T, will cauſe an Augmentation or Diminution of the magnifying Power of the Teleſcope.

734. Therefore to be able to find, in all Caſes, what this Power is, it will be neceſſary to trace the *diverging Rays* from any Point in the Image (*i m*), now conſidered as an Object, through all the Lenſes, till they unite again in the Image NO. In order to this, it may be obſerved that in all theſe Teleſcopes the Diſtance of the Image (*i m*), from the Lens R, is always leſs than the focal Diſtance of that Lens, and that therefore

diverging Rays will, after Refraction, ſtill diverge, or have a negative Focus, becauſe, in this Caſe, the Theorem will be $\frac{dr}{d-r} = -f$, (by 434.)

735. Let the Diſtance of the Lens S from R be called s, and then $f + s = D =$ the New Diſtance of the Radiant Point from the Lens S, whoſe focal Diſtance let be R; then $\frac{DR}{D-R} = F$, the Diſtance of the conjugate Focus to D; which (becauſe D is greater than R) is poſitive, or the Rays, after Refraction through S, are converging to a Point on the oppoſite Side from D.

736. Let the Diſtance between the Lenſes S and T be called t. Then $F - t =$ D, the Diſtance of the Radiant Point from the Lens T, whoſe focal Diſtance we now call R; and ſince the Rays are now *converging*, or D negative, we have (by 441) the Theorem $\frac{\mathrm{DR}}{\mathrm{D+R}} = \mathrm{F} =$ the focal Diſtance by all the three Lenſes, where the Rays again unite, and the Image *NO* is formed of the firſt (*i m*).

737. Now in any Lens, or Combination of Lenſes, the Object is in Proportion to its Image as its Diſtance from the Lens (or Lenſes) is to the Diſtance of the Image from the ſame, as was ſhewn (*Art.* 30). Therefore $im : NO :: d :$ F. Which Ratio is eaſily found for any given Values of r, R, R, d, s, and t.

738. Thus for Example, let $r = 2$, and $d = 1$; then $\frac{d-r}{dr} = -2 = f$. Therefore if $s = 1{,}5$, then $f + s =$ D $= 3{,}5$; and let R $= 2{,}5$, then $\frac{DR}{D+\mathrm{R}}$

$\frac{DR}{D+R} = 8{,}75 = F$. Laſtly, put $t = 3$, and $R = 3$, and we have $F - t = D = 5{,}75$. Then $\frac{DR}{D+R} = F = 2$, very nearly. Therefore the Image $im : NO :: d : F :: 1 : 2$.

739. If all the Lenſes R, S, T, are of the ſame focal Diſtance, and their Intervals equal to that focal Diſtance, then will $d = F$, or $im = NO$, or the Image (*i m*) will be only rectified, but not magnified. And this is very nearly the Conſtruction of the *Achromatic*, and all other Six-Glaſs Teleſcopes, as they are now made.

740. I might now proceed to ſhow how a Refracting Teleſcope of this Sort, may be conſtructed with a proper Diſpoſition of the four Glaſſes Q, R, S, T, ſo as to render it capable of the following Properties. (1.) That what is commonly called an *Achromatic Teleſcope*, ſhould be *truly and really ſuch*, whereas at preſent it is not ſo. (2.) That it ſhould admit of *many different magnifying Powers*; though, as now made, it has but *One*. (3.) That it ſhould exhibit a *perfect View of all Objects nigh at Hand*, and thus become an *Engyſcope*. (4.) That it may be uſed as a *Megalaſcope* to view *Flowers*, *Minerals*, *Ores*, *Intalios*, *Medals*, &c. to the greateſt Advantage. (5.) That it may be uſed as a *Microſcope*, for very *ſmall Objects*, as it will magnify them 20, or 30 times in Diameter. (6.) That it ſhall exhibit the moſt perfect Picture or Landſcape-View of Objects, in juſt Perſpective, that can be effected by Inſtruments, and without any Trouble at all. (7.) That it is the beſt Sort of Inſtrument for viewing Prints in Perſpective, and giving them their natural *Relievo*. ——— But I

fear too much may be eaſier ſaid than too little upon thoſe Subjects; and it may be better to verify theſe Properties of an Universal Refractor by Experiments, than by a prolix mathematical Demonſtration.

CHAP. VII.

The Theory and New Construction of a Solar Microscope for procuring a great Increase of Light, and Perfection in the Field of View, and the Images of Objects therein.

741.

I Have already given one new Conſtruction of a Solar Microscope, *viz.* for repreſenting Objects in the magnified *Diſk of the Sun* (549). But when the Images of ſmall Objects are required to be formed of an enormous Size, this Method will not do, as a great Diſtance of the Image from the Glaſs, and a proportionable Degree of Light, will be neceſſary upon that Occaſion.

742. Now the common *Solar Microſcope* having an illuminating Glaſs about one Inch and half Diameter, gives Light full little enough to ſhew the Images of Objects at the Diſtance of 16, 18, or 20 Feet, (made by a Glaſs of $\frac{1}{10}$ of an Inch focal Diſtance) for then the Light which is con-

tained in a Circle of $\frac{1}{10}$ of an Inch upon the Talk, will, by Refraction thro' the ſaid Object-Lens, be diffuſed over a Circle 20 Feet in Diameter, in a Room 20 Feet wide; conſequently the Diminution of Light will be in the Ratio of 5760000 to 1, or the Image will be ſo much leſs enlightened than the Object. Nay, even with a Glaſs $\frac{1}{20}$ of an Inch, the Light will be diminiſhed near 2600000 times.

743. The Section of the Cone of Rays $\frac{1}{10}$ of an Inch Diameter will have the Sun's Rays condenſed 225 times; the Diameter of the Illuminator being $1\frac{1}{2}$ Inch focal Diſtance. Hence the Light of the Image made by a Lens $\frac{1}{10}$ of an Inch at the Diſtance of 20 Feet, is to that of the common Sun-Beams, as 225 to 5760000, or as 1 to 26000 nearly.

744. Hence it appears that the greateſt Thing wanted for an Improvement of this Inſtrument is an Increaſe of Light upon the Object, or its Image, to ſhew it more plainly and perfectly. If we attempt this by only encreaſing the Diameter of the Illuminator in the Frame, the Heat in the Section of the Cone of $\frac{1}{10}$ of an Inch will be ſo great that no Sort of tranſparent Objects will endure it. Indeed, in a Glaſs of the uſual Size, thoſe tender Objects are never brought into ſo denſe a Part of the radious Cone, or ſo near the Focus as we have ſuppoſed; and therefore the Images are ſtill leſs enlightened than in the Ratio above-mentioned.

745. Let us therefore have Recourſe to a Section of the radious Cone = $\frac{1}{20}$ of an Inch; and then if we uſe a Lens $\frac{1}{20}$ focal Diſtance, the Image of that Section, or Circle of Light, will ſtill be the ſame as before at the ſame Diſtance,

ſtance, and of Courſe the Quantity of Light diffuſed over it will alſo be the ſame; and the Intenſity of Heat will be as great as moſt Sorts of Objects will bear, when the Light is encreaſed in any conſiderable Degree.

746. From what has been ſaid, it is evident that there is no way of getting an Encreaſe of Light but by enlarging both the Diameter and focal Diſtance of the Illuminator; and then in equal Sections of the radious Cones, the Light will be encreaſed in the duplicate Ratio of the Diameter or focal Diſtance of the ſaid Illuminator. But then the Queſtion is, how this is to be done? For in the common Conſtruction, if the Illuminator be enlarged to twice the Diameter, and twice the focal Diſtance, then the Looking-Glaſs, Tubes, *&c.* muſt all be encreaſed to twice the Length and Breadth; and ſo the Bulk, Weight, and Expence of the Inſtrument would become intolerably great, and thereby render this Method impacticable.

747. There is only one other Way that I can think of encreaſing Light with any Degree of Conveniency, and to give a proper Idea of what I mean, I ſhall repreſent it in the Section of the Solar Microſcope made upon this New Conſtruction, as ſhewn in *Fig.* 14 of *Plate* VIII. The Plane is ſuppoſed to be directly South; the Sun in the Meridian, about the Equinoctial Day.

748. Then let AB be the Frame or Plate of the Microſcope; CD the Hole in the Middle, into which the Tube is ſcrewed on one Side, and the illuminating Glaſs is on the other; the Size of this Illuminator is commonly $1\frac{1}{2}$ Inch Diameter, but that not giving ſufficient Light, we ſuppoſe it removed, and another much larger Lens GH

placed at the End of the Reflector (or *Looking-Glaſs)* EF, moveable upon a Foot at F. We may ſuppoſe this Glaſs GH to be from 3 to 6 Inches Diameter, and therefore, in our Example, let it be a Lens of 4 Inches Diameter, and its focal Diſtance 12 Inches.

749. Then the Looking-Glaſs EF being properly adjuſted, and the Lens GH ſo poſited as to receive the Rays of the Sun *a*, *b*, *c*, in a perpendicular Direction, they will be refracted thro' it tending towards a Focus in the Axis IL at 12 Inches from the Lens. But this Cone of Rays falling upon the Glaſs EF, between K and L, will, by it, be reflected to a Focus O in the reflected Axis L*g*, which is parallel to the Horizon.

750. Now the Focus O of any Lens being the Image of the Sun, will of Courſe be a Circle, and in the preſent Caſe of a Lens 12 Inches focal Diſtance, this Circle will be extremely near $\frac{1}{10}$ of an Inch Diameter, by allowing $30'$ for the Diameter of the Sun. Hence the Vertex N of the Cone will be $\frac{3}{10}$ of an Inch beyond the Focus O. Therefore the Diameter of the Section of the Cone $\frac{3}{10}$ within the Focus, will be $\frac{2}{10}$ of an Inch; at $\frac{6}{10}$ within the Focus, the Section will be $\frac{3}{10}$; at $\frac{9}{10}$ from the Focus the Section will be $\frac{4}{10}$; and at $1\frac{2}{10}$ from the Focus the Section will be $\frac{5}{10}$, or Half an Inch in Diameter.

751. Therefore at $\frac{3}{20}$ of an Inch from the Focus the Section of the Cone will be alſo $\frac{3}{20}$ of an Inch in Diameter; and it is found by Experience that nearer the Focus O no Objects can be placed. Therefore the firſt Inch and Quarter of the radious Cone may be applied for all the various Powers of magnifying *Microſcopic Objects*;

and from thence, through the next Inch, the *Megalaſcope* may be applied in the greateſt Perfection.

752. Let Z be the Center of the Hole CD in the Frame, then ZO will be about $5\frac{1}{2}$ of the 12 Inches, and conſequently long enough for a Movement of the Microſcope and Megalaſcope thro' the 2 Inches juſt mentioned. Hence it will appear that in this Conſtruction of the *Solar Microſcope*, a much leſs Apparatus of Tube-work will be neceſſary than in the common Sort, where DOC makes all the Cone, and the Length Z *g* is 7 or 8 Inches.

753. If we put the Diameter of the common Illuminator $CD = 1{,}5$, and the Diameter of the Lens $GH = 3$, then, in the ſame Section RS of the Cone produced by each Lens ſeparately, we have the Intenſity of Light as CD^2 to GH^2, that is, as 2,25 to 9, or as 1 to 4. If $GH = 4$, then the Ratio of the Increaſe of Light will be that of 2,25 to 16, or as 1 to 7,1. Suppoſe $GH = 5$; then 2,25 : 25 :: 1 : 11 nearly. So that by this Conſtruction you may encreaſe the Light upon Objects, or their Images, at leaſt *ſeven times* with Eaſe, or ten times with very little Trouble or Expence.

754. If the Holes in Sliders containing the Objects RS be equal to the circular Section of the Cone, and the focal Diſtance of the Object Lens be equal to the Diameter of ſuch Section or Hole; then will the Experiments ſucceed in the moſt perfect Manner; for it is evident the Image of the Hole in the Slider will be the *enlightened circular Field of View* upon the oppoſite Side of the Room; and all the Objects RS contained between the Talks will have their Images TV di-

ſtinctly repreſented, and much brighter than in the common Way.

755. Again, in the Uſe of every Magnifier, this *Field of View*, or *illumined Circle*, is conſtantly the ſame, and equal to the *Width of the Room* in Diameter. Becauſe it is evident d f : RS : : d W : T V, but by Suppoſition d f = RS nearly, therefore the Diameter T V of the illumined Circle is equal to the Width d W of the Room.

756. It is further evident, that becauſe in every circular Section of the Cone RS there is the ſame Quantity of Light, therefore the Field of View will be equally illumined by each ſeveral Magnifier PQ, if the above-mentioned Conditions (754) are obſerved; and alſo that the Apertures of all the Lenſes are proportioned to their focal Diſtances.

757. In Lenſes of different focal Diſtances, the linear Dimenſions of the Images will be inverſely as the ſaid focal Diſtances, if theſe be as 1, 2, 3, 4, 5, then the *Length* and *Breadth* of the Images will be as 5, 4, 3, 2, 1; and their Areas or Surfaces as 25, 16, 9, 4, 1; and their Bulks or Solidities as 125, 64, 27, 8, 1. So that a Lens of $\frac{1}{10}$ of an Inch focal Diſtance will magnify a *Mite*, a *Flea*, *&c.* 125 times as much as a Lens whoſe focal Diſtance is $\frac{5}{10}$ or $\frac{1}{2}$ an Inch. But this laſt, in a Room 20 Feet wide, will magnify the Image of a *Mite*, *&c.* 480 times in Length and Breadth; a Lens therefore of $\frac{1}{10}$ of an Inch will magnify it 5 times more, or 2400 times; ſo that if a Flea be $\frac{1}{10}$ of an Inch long, its Image will be 20 Feet long upon the Side of ſuch a Room, if large enough to receive it. Then ſay, as $1^3 : 2400^3 :: 1 : 13824000000$; whence it appears that the Flea is magnified 13

Thouſand 824 Millions of times in Bulk. A magnifying Power beyond Conception great!

758. Since the Superficies of the Images are *inverſely* as the Squares of the focal Diſtances of the Lenſes, and by the above Conſtruction, the Intenſity of Light (or Denſity of Rays in the Cone) in that Part where the Object is placed is alſo inverſely as the ſaid Squares of the focal Diſtances; therefore the Light upon the Objects will be *directly* as the Areas or Surfaces of the Images, and conſequently thoſe Images, by all the Lenſes, will be equally luminous or bright.

759. When the Aperture of the Sliders and Lenſes are properly adapted to each other, the Field of View will be equally large, luminous, and well defined in the extreme Parts; and if the Lenſes are *Plano-convexes*, they muſt have their plain Sides turned toward the Object; but by the Theory, as well as Experience, it will be found, that when the focal Diſtance is not very ſmall, it is better to make it by *two double and equally convex Lenſes* placed together in the ſame Cell. And theſe Precautions and Punctilios obſerved, will make this New Conſtruction of a Solar Microſcope (by Glaſſes) the moſt perfect of any I know of at preſent. For what relates to SOLAR MICROSCOPES by REFLECTION only (*without any Glaſs at all*) I ſhall refer the Reader to my OPTICAL ESSAYS, lately publiſhed, where he will find this, and many other curious Subjects, treated of in no other Book.

POSTSCRIPT.

The DESCRIPTION *of a New* Univerſal MICROSCOPE, *with all the Uſes of* SINGLE, COMPOUND, OPAKE, AQUATIC, *&c.* *Microſcopes.*

I Suppoſe it will not be queſtioned, that ſuch a Conſtruction of a Microſcope, in which all the Uſes of Glaſſes may be found, which have hitherto been conſidered as peculiar to the SINGLE, the COMPOUND, OPAKE, AQUATIC, *&c.* MICROSCOPES, may with Propriety be eſteemed an *univerſal* One. And that which I here offer the Public is propoſed to anſwer that Character, and which I ſhall now proceed to deſcribe, in all its Particulars.

The COMPOUND MICROSCOPE is ſuch an one as produces its Effect of magnifying ſmall Objects by a Plurality or Compoſition of Glaſſes, to anſwer ſome advantageous Purpoſes which are not obtainable by a *Single Microſcope*; and to avoid ſome Inconveniencies to which they are liable. That Form of a Compound Microſcope which, by Experience, is found to anſwer every Purpoſe and Mode of magnifying Objects in the beſt Manner, conſiſts of the following Parts, *viz.*

Fig. I. ABC is the Body of the Microſcope.

D is a Joint, by which it is moveable *vertically*.

E is a hollow ſquare Socket, with

F a Screw, by which it is fixed to the Part at D.

DQR is a ſtrong Braſs Pillar, or Stand.

S, T, V, the *Tripod*, or three Feet on which it ſtands.

GHI is a Stage on which Objects of different Sorts are placed to be viewed.

K is a ſtrong Screw by which the Stage is rendered moveable horizontally.

M, N, are two Braſs Sockets, connected by an adjuſting Screw, and moveable up and down upon the ſquare Part of the Stand.

O is a Screw for fixing the Socket M.

P is a long adjuſting Screw by which the Socket N is moveable, and the Objects upon the Stage adjuſted to the View.

W is a concave Mirror, or Speculum, fixed at

X, juſt under the central Part of the Stage, for illuminating traſparent Objects.

Y is a concave Lens moveable at

Z, in a ſpring Socket; by this Lens opake Bodies are ſufficiently enlightened for the View.

To be more particular, the Glaſſes, or ſmall Magnifiers contained in braſs Cells, to be ſcrewed upon the Pipe at A, are in Number Six, as repre- *Fig.* II. ſented in the Plate, and their focal Diſtances expreſſed in *Tenths of an Inch*, are as near as we can get them to the following Numbers, *viz.* 1, 2, 3, 5, 7, 10. And the two Glaſſes at B and C do together magnify equally to a ſingle Lens of about $1\frac{1}{4}$ Inch. Alſo between the Object Glaſs, and this ſingle equivalent Eye-Glaſs there is about 5 Inches Diſtance; conſequently by the Theory of compound

Microſcopes (explained in the foregoing *New Elements of Optics)* the magnifying Powers in reſpect to the *Length* and *Breadth* of Objects, will be nearly expreſſed by the following Numbers for Lenſes from 1 Inch to $\frac{1}{50}$ of an Inch focal Diſtance. But it muſt be remembered that in this Computation we put 8 Inches for the Limit of diſtinct Viſion by the naked Eye.

		Tenths of an Inch.	Mag. Power.	
Lenſes 1	focal Diſtance	*10	32	times.
2	—— ——	9	35	
3	—— ——	8	40	
4	—— ——	*7	$45\frac{3}{4}$	
5	—— ——	6	$53\frac{1}{2}$	
6	—— ——	*5	64	
7	—— ——	4	80	
8	—— ——	*3	107	
9	—— ——	$2\frac{1}{2}$	128	
10	—— ——	*2	160	
11	—— ——	$1\frac{1}{2}$	214	
12	—— ——	*1	320	
13	—— ——	$\frac{1}{20}$	640	
14	—— ——	$\frac{1}{30}$	960	
15	—— ——	$\frac{1}{40}$	1280	
16	—— ——	$\frac{1}{50}$	1600	

The Lenſes which have an Aſteriſm (*) prefixed are thoſe we generally put into this Microſcope; though it is impoſſible to be always certain of the focal Length of Lenſes, even though they are ground in the ſame Tool, as different Sorts of Glaſs have unequal Refractions, and therefore with the ſame Degree of Convexity will have various focal Diſtances. But the Numbers above are near the Truth in general. It will be found difficult to uſe Lenſes in a compound Microſcope eroſcope of more than $\frac{1}{20}$, or at moſt, $\frac{1}{30}$ Part of an Inch; nor can we uſe theſe to any Purpoſe without a great Increaſe of Light. The moſt judicious and experienced Opticians and Naturaliſts ſeldom uſe very deep Magnifiers, as knowing, after all that can be ſaid, that a moderate or mean Degree of magnifying is, in general, by much the beſt for natural Enquiries and Reſearches.

This Compound Microſcope is in the beſt Manner adapted to view *tranſparent Objects*; for if they are ſuch as can be put into the *concave Glaſs* in the Middle of the Stage at H, then they will be ſufficiently enlightened by the Reflector W below, in one Side of which is a plain Speculum, and in the other a concave one, as both Sorts are occaſionally neceſſary.

If the tranſparent Objects are ſuch as may be included between *Talks* in the *Ivory Sliders*, then
Fig. III. there is a Part ABCD, called the *Slider-holder*, which is fitted to the Hole at H, in the Middle of the Stage, and in which all the Variety of Objects in Sliders may be viewed to great Perfection, by reflected Light from the Speculum W below.

As ſome curious Experiments with tranſparent Objects require the Light to be very pure, and adjuſted to a proper Degree, there is an inverted
Fig. IV. Cone of Braſs, ABC, to be placed in the under Part of the Hole at H by its broad End or Baſe AC; and by the narrow End B only the interior and purer Light contained in the upper and denſer Part of the large Cone of Rays reflected by the Speculum W, can illuminate the Objects to be viewed. This Cone is indeed of more immediate Uſe in the *Single Microſcope*, to be mentioned

tioned by and by. Thus, it is plain, in one or the other of theſe Ways all Kinds of *tranſparent Objects* are to be viewed in the utmoſt Perfection.

In this Conſtruction it is alſo as evident that every *opake Object* may be ſhewn as well in this as in that which is uſually called the *Opake Microſcope*, becauſe here is all the ſame Apparatus for that Purpoſe, and much more; for this is both a *Single and Compound* OPAKE *Microſcope*.

Thus, if any opake Object be laid upon the Glaſs at H, it may be very ſtrongly illuminated by the Lens Y, moveable higher or lower in the Socket Z, to make the Light upon the Object greater or leſs, as Occaſion requires. In this Caſe you have the Advantage of a large and delightful *Field of View*, and Objects of all Shapes and Sizes are immediately viewed upon the Stage GHI, as it is ſo eaſily moved up and down by the ſliding Sockets M and N; faſtened in any Poſition by the Screw O; and adjuſted for the moſt accurate Inſpection by the Screw P.

In many Caſes it may be requiſite to view light-coloured Objects upon a dark Ground, and the contrary; therefore to anſwer ſuch Purpoſes there is provided a round flat Piece of Ivory, with one Side white and the other black, fitted into the Hole a G, and to be taken in and out at Pleaſure.

To anſwer theſe Purpoſes ſtill more generally, there is a Pair of Plyers, AB, moveable in a braſs Spring Socket at C, and by the Shank at D it is fitted in the Hole of the Stage at I, where it has a horizontal Motion, and alſo a vertical Motion (up and down) by its Joint at E. By the Pincers at the End A any Object may be very readily adjuſted to the View, and illumined *Fig.* V.

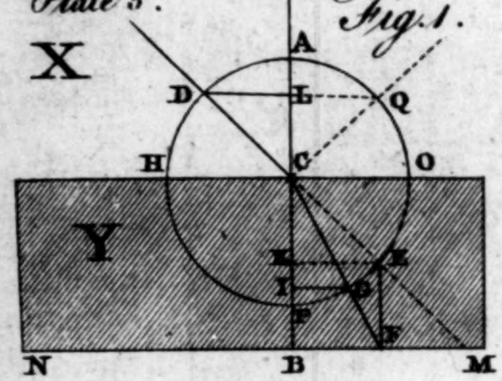
by the Lens Y. Alſo at the other End of the Plyers B, there is a ſmall cylindric Piece of Ivory F ſcrewed on, with one End black, the other white, for the above-mentioned Purpoſes.

But often times Objects will be found which require the Uſe of the Plyers AB, and are ſeen to the greateſt Advantage by Light reflected upon them; for which Purpoſe a ſmall concave metallic Speculum AB is ſcrewed into the End *Fig.* VI. CD of a Tube CDEF, which is made to go on upon the Pipe of the Microſcope; and the large Cone of reflected Light from the Mirror W will paſs through the Hole H to this ſmall Concave AB, by which it will be reflected upon the Object in the Plyers at A, but more eſpecially upon the Ivory at F, where exceeding ſmall Objects require the greateſt Degree of Light they can bear.

Further; to make this Microſcope anſwer all the Ends of a *Single Opake Microſcope*, there is a *Fig.* VII. braſs Piece AB, with a ſquare Hole or Socket, *Fig.* I. to go on upon the Shank at D, when the Body of the Microſcope ABC is taken off, and is there made faſt by the Screw C. To this is annexed a ſtrong braſs Ring DE, into which are ſcrewed the ſame *Liburkuns*, as they are called, or Concaves with a ſmall Lens in the Middle of each, as are uſed in the *Single Opake Microſcope*; and being applied to Objects on the Stage GHI, or *Fig.* V. in the Plyers AB, they are viewed in the ſame Manner here, as they are there, with this additional Advantage, that in the preſent Microſcope the Light is much more intenſe from the Speculum W, than the common Light but once reflected, in the uſual Form of this Inſtrument. See *Fig.* 13 in the Plate of my OPTICAL ESSAYS.

Laſtly;

Plate 5.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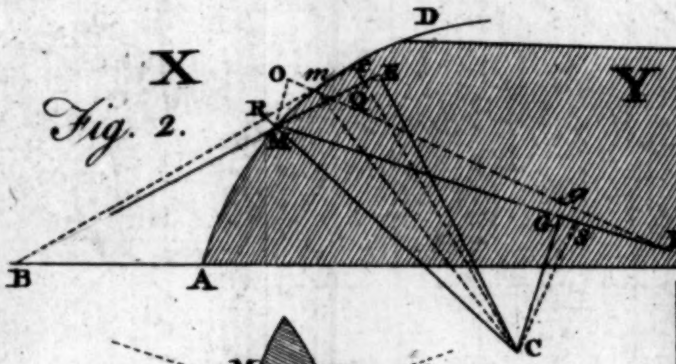
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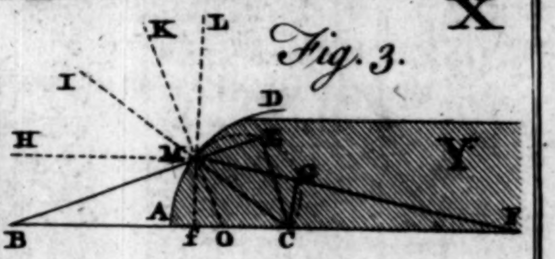
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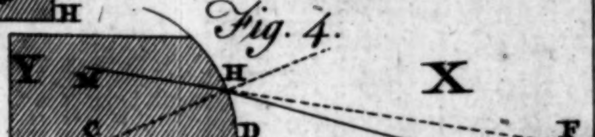
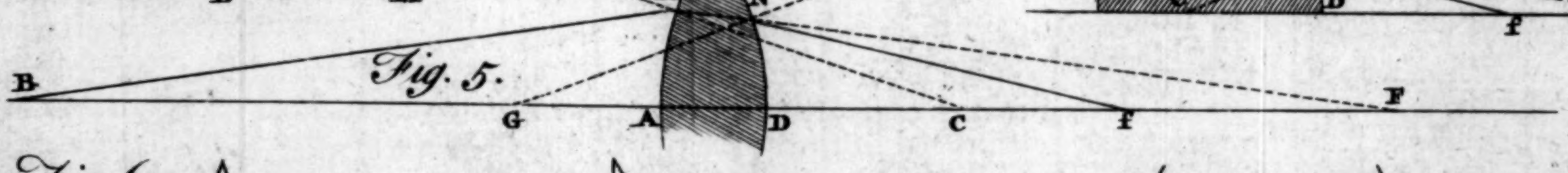


Fig. 4.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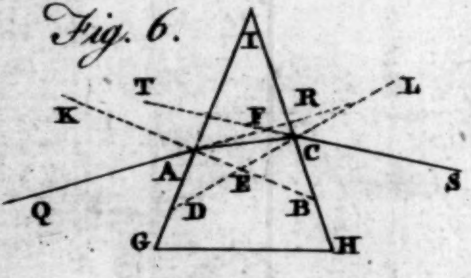
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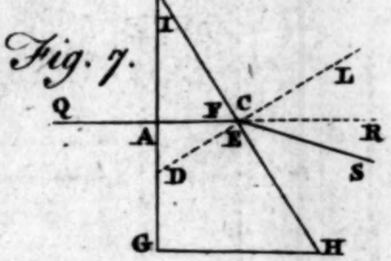
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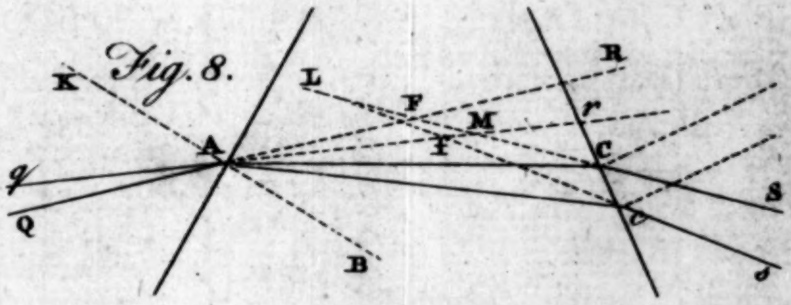
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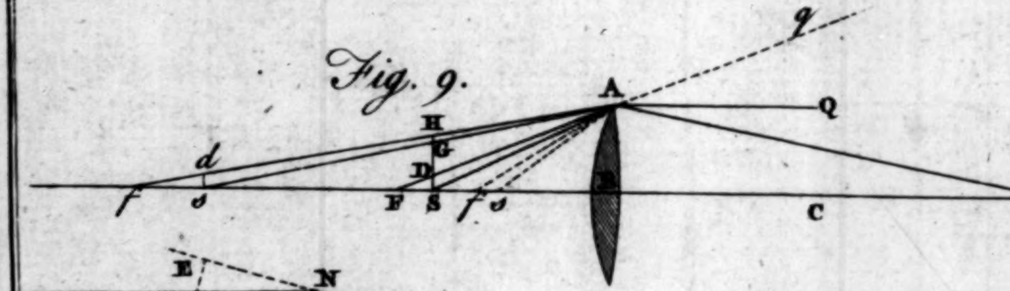
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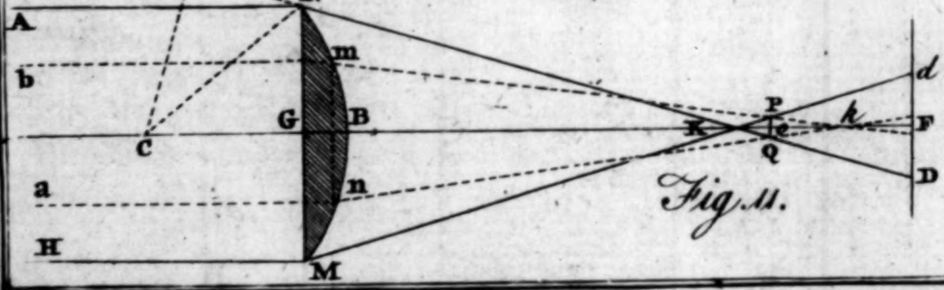
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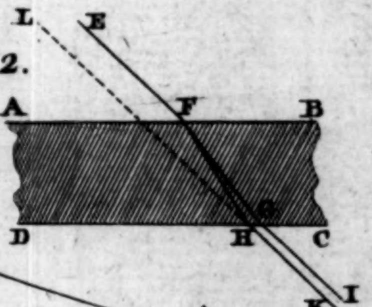
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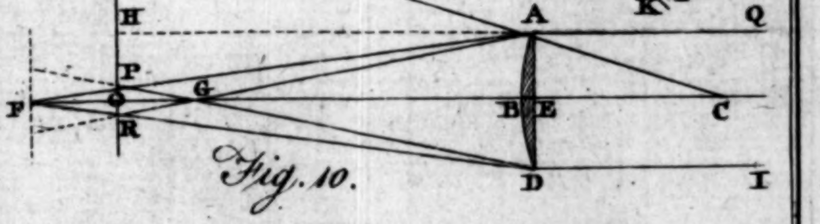
Fig. 10.

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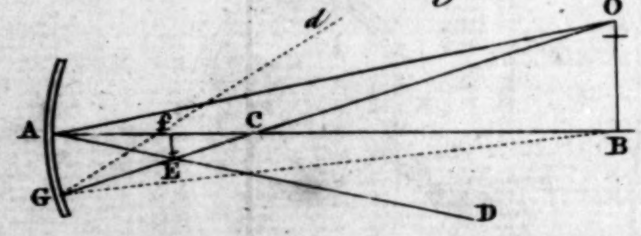
Plate VI.



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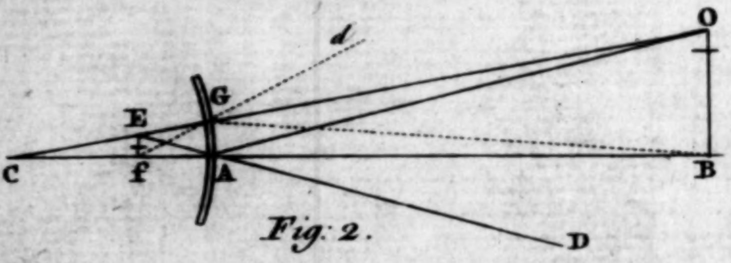

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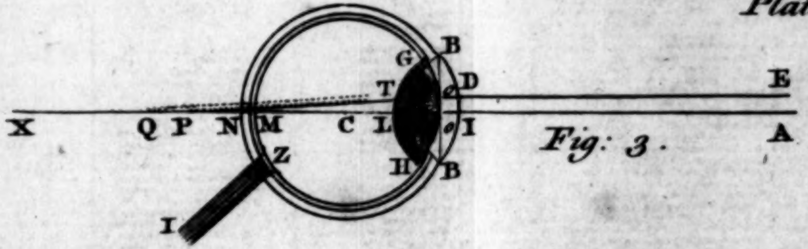

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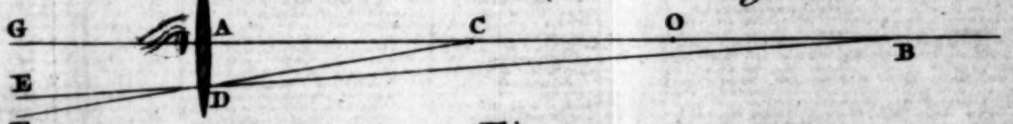

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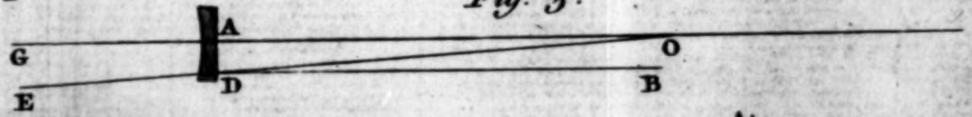
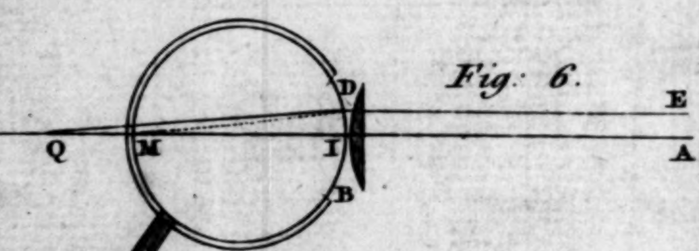


Fig: 5.



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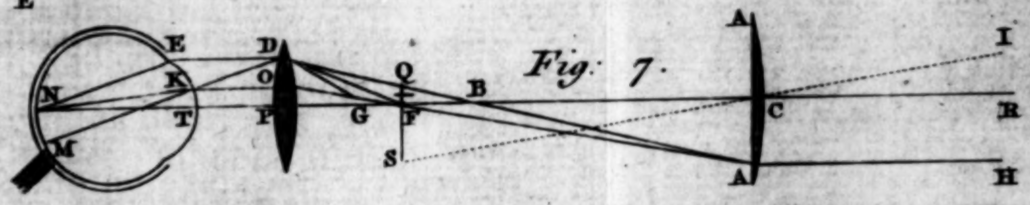

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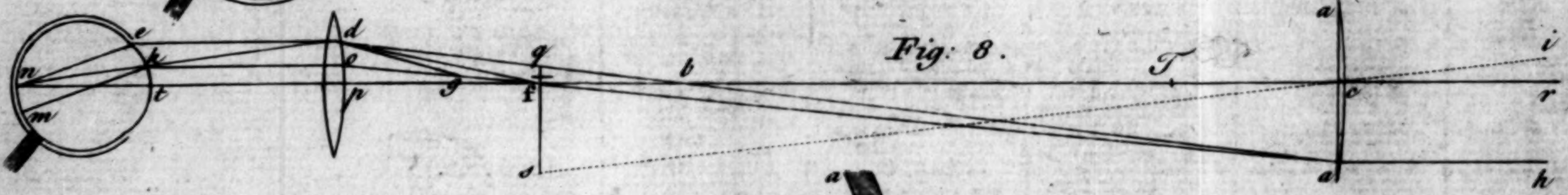

Fig: 8.

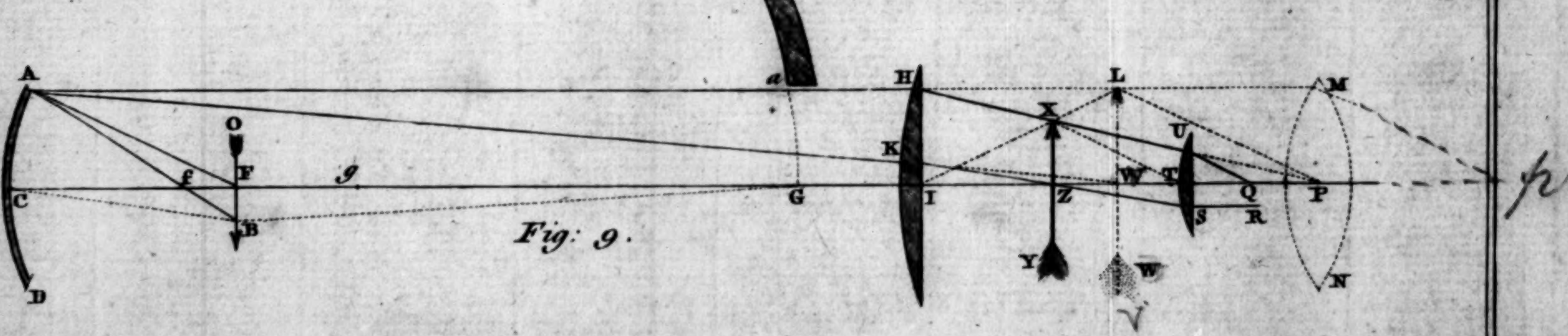

Fig: 9.

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Plate VII.

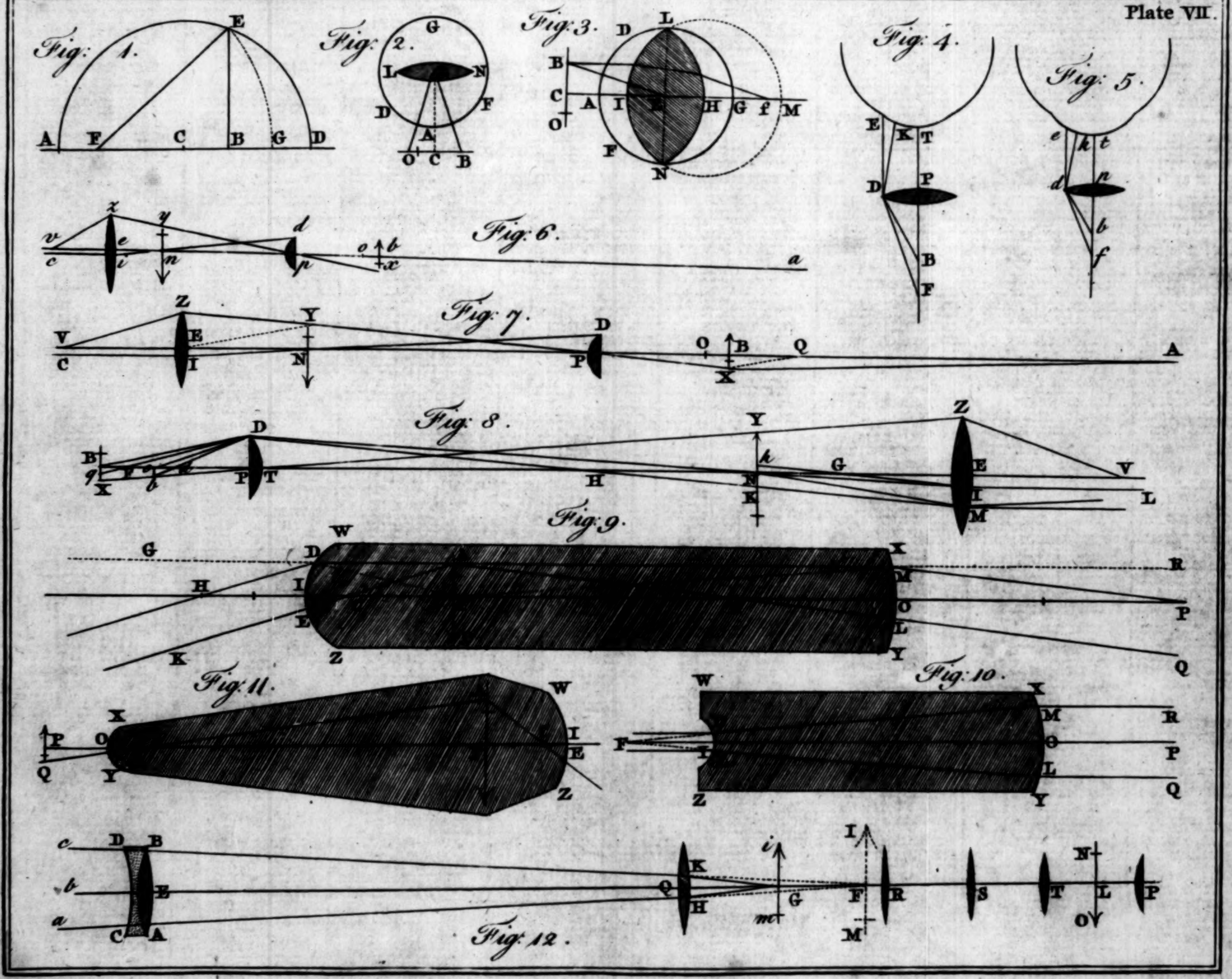
Fig: 1.
E
A F C B G D
Fig: 2.
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Fig: 3.
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Fig: 4.
E K T
D P
B
F
Fig: 5.
e k t
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Fig: 6.
z y d
v e
c i n p
o b x a
Fig: 7.
Z Y D
V E
C I N P
O B Q A
X
Fig: 8.
D Y Z
B k G E V
q F b a P T H N I L
X K M
Fig: 9.
W X
G D R
H I M
E O P
K L
Z Y Q
Fig: 11.
W
X I
P O E
Q Y Z
Fig: 10.
W X
M R
F O P
I L
Z Y Q
c D B
b E
a C A
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I F R M
S T
N L P O
Fig: 12.

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Plate VIII.

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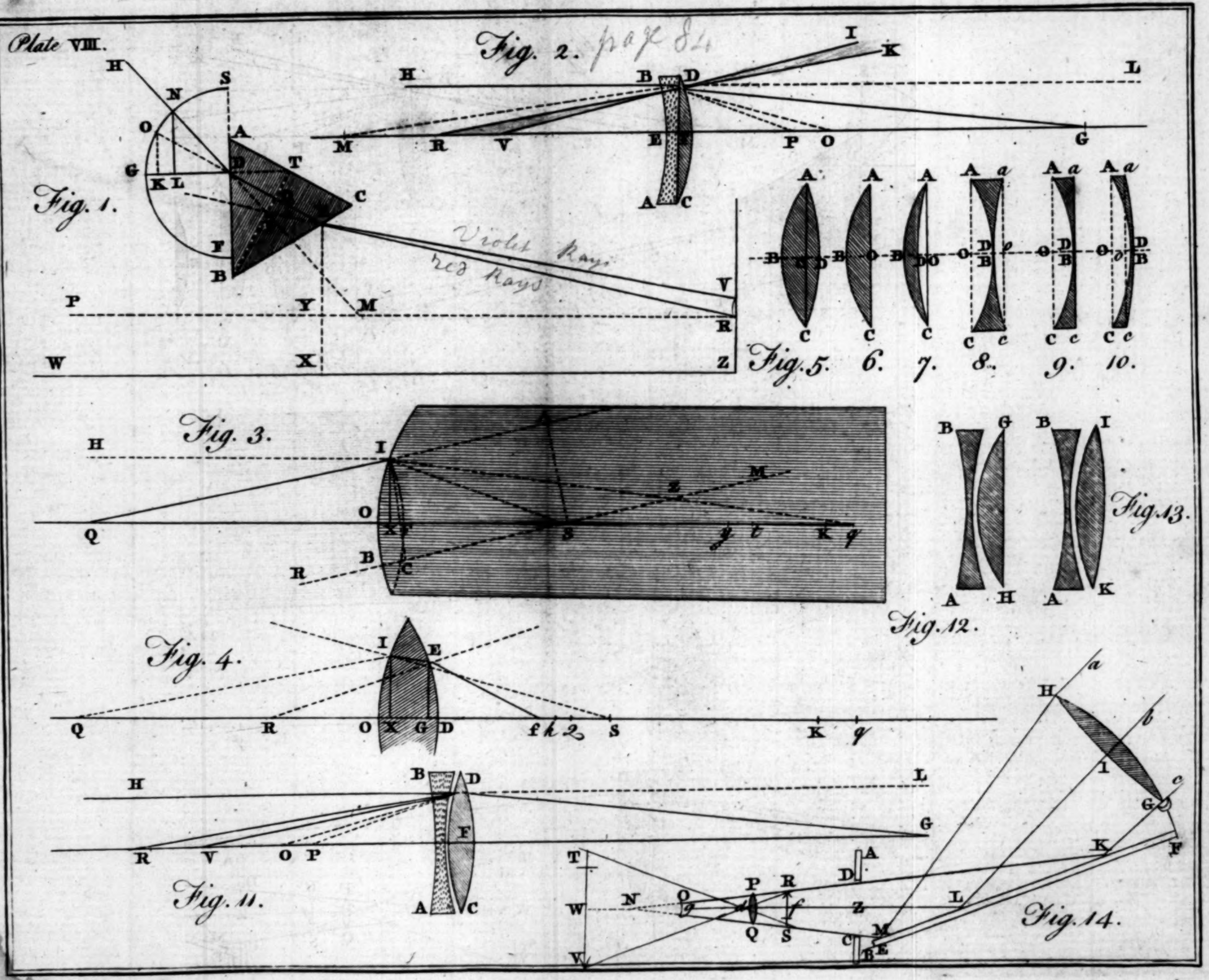

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5. 6. 7. 8. 9. 10.

Fig. 11.

Fig. 12.

Fig. 13.

Fig. 14.

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Laſtly; every Thing ſhewn in the AQUATIC MICROSCOPE is to be ſeen equally in This; becauſe, ſince the Hole at H in the Stage, is very large, it will admit of a Concave ſufficiently capacious for any Purpoſes of viewing Objects in Water, of any Sort whatſoever. And not only the Magnifiers, but the Mode of applying them, is very nearly the ſame here as in thoſe of the common Form. But in this Conſtruction, you have both the Single and Compound Aquatic Microſcope, for the Stage GHI being moveable horizontally, and the Magnifier at A moving vertically on the Joint at D, 'tis plain, every Part of the Water in the concave Glaſs at H may be brought under it, and the moſt minute Objects well enlightened by the Concave AB, and ſhewn with great Diſtinctneſs.

The other Parts of the Apparatus are common to all Microſcopes. Every Body knows the Uſe of the *Ivory Sliders*, for holding and applying tranſparent Objects as above directed; ſee its Form in *Fig*. 8. But in *Fig*. 9. you have the Form of a braſs Slider, with ſeveral ſmall glaſs Concaves fixed in one Side, and over them a Slip of clear plain Glaſs is made to ſlide in the Frame, and thereby to confine in the Hollow of the Glaſſes very ſmall living Objects, as a *Flea*, *Louſe*, *Mite*, *&c* and prevent their crawling out of the Field of View.

There is alſo what is called a BUG-BOX, conſiſting of two Parts; the loweſt contains a large Concave, and the upper Part contains a plain Glaſs, which being ſcrewed upon the Concave, will confine any larger Animal, as a *Bug*, an *Ant*, a *Spider*, a ſmall *Fly*, *&c*.

As the Circulation of the Blood is one of the

nobleſt

nobleſt Experiments of the Microſcope, ſo ample Proviſion is made for it by a Set of GLASS TUBES of different Sizes for applying the tranſparent Parts of proper Objects for that Purpoſe, ſuch as ſmall *Fiſh*, *Tadpoles*, and *Water-Neuts*, the beſt Subject of all; ſuch a Tube is AB. It being neceſſary to ſtop the open End B, when the Animal is in, with a Cork, there is a ſmall Hole at the other End A to give Air to the Animal. Theſe Tubes are applied to the Hole H in the Stage by two Steel Springs on the under Part, bent to receive them. *Fig.* X.

In Caſe it be required to view the Circulation in the Tail of a larger Fiſh, as a *Gudgeon*, *Loach*, *&c.* there is an Inſtrument of Braſs called the FISH-PAN, contrived of a proper Form to hold and confine it; (ſee *Fig.* X.) where ABD is the incurved Plate to receive the Body of the Fiſh; CFG is a Ribbon to tye the Fiſh to the ſaid Plate or Pan, and is kept tight by a Spring behind at H. At the End AD, is a long tranſverſe Hole or Slit, over which the tranſparent Tail of the Fiſh is placed, and then by the Shank at E, on the under Side, it is put upon the Stage through a Hole at I, and there eaſily adapted to the Magnifier A, by moving it every Way under the ſame. *Fig.* XI.

It is often required to ſee what many ſmall Objects are, and how they are beſt diſpoſed in Sliders, Glaſſes, Tubes, *&c.* for which Purpoſe there is a Hand-magnifier ABC, containing a Lens of about one Inch focal Diſtance, to be uſed upon all ſuch Occaſions. *Fig.* XII.

Beſides the above Particulars there is a Pair of NIPPERS (or *Forceps*) to take up ſmall Objects, in order to place them on the Stage, between the

Talks, *&c.* alſo a *Camel-Hair* BRUSH, for cleanſing Talks, Glaſs, *&c.* A ſmall Wire, with a ſpiral Screw at the End, for holding Cotton, *&c.* for cleanſing the glaſs Tubes. A little *Ivory Box* with ſpare Talks, and Wires to faſten them in the Sliders. A Piece of *Shammy Leather* is uſeful upon all Occaſions for wiping the Glaſſes of every Sort, as it will cleanſe them well without hurting their Surfaces.

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I have ſaid nothing here of the CATOPTRIC MICROSCOPE, or that by REFLECTION, as the Theory thereof may be ſeen in the *New* ELEMENTS of OPTICS, and its practical Uſes are deſcribed in my *Optical* ESSAYS; as alſo of a MICROMETER, which will give the Dimenſions of the ſmalleſt Objects to the 1000dth Part of an Inch.

FINIS.

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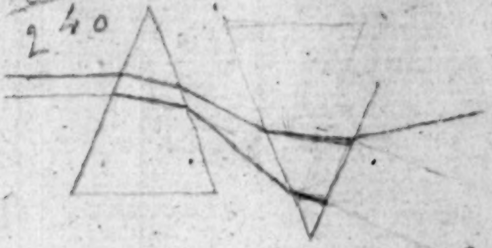

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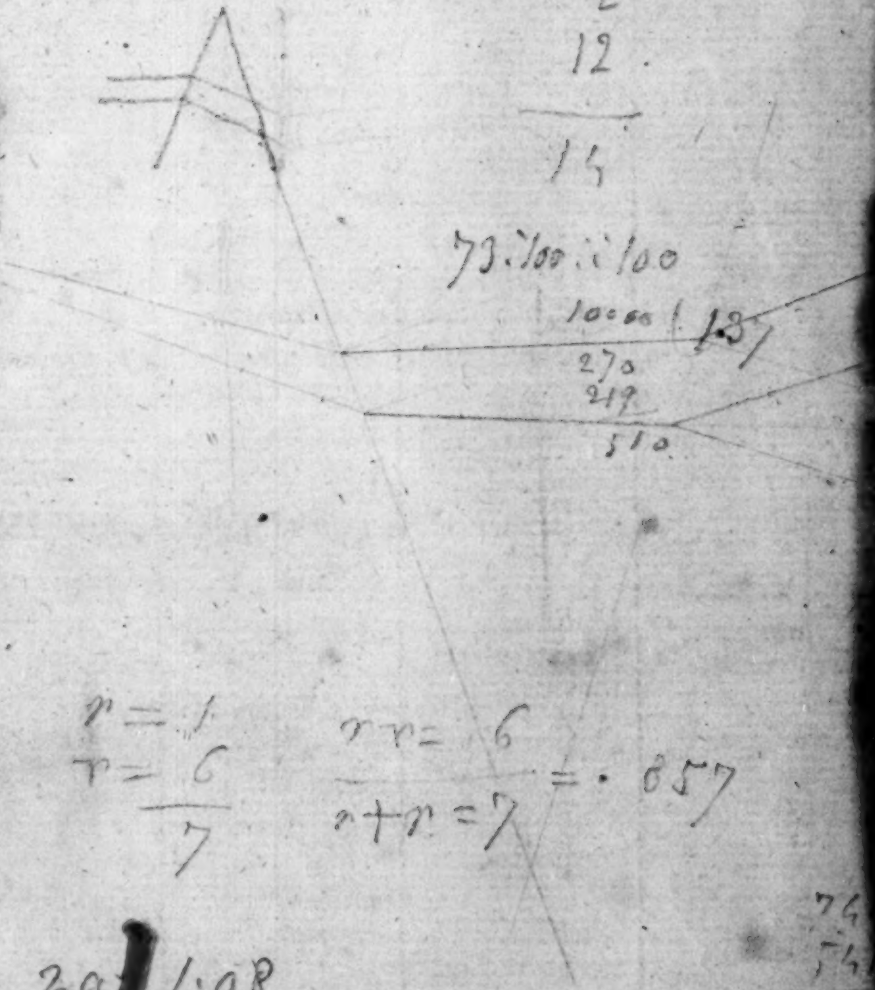

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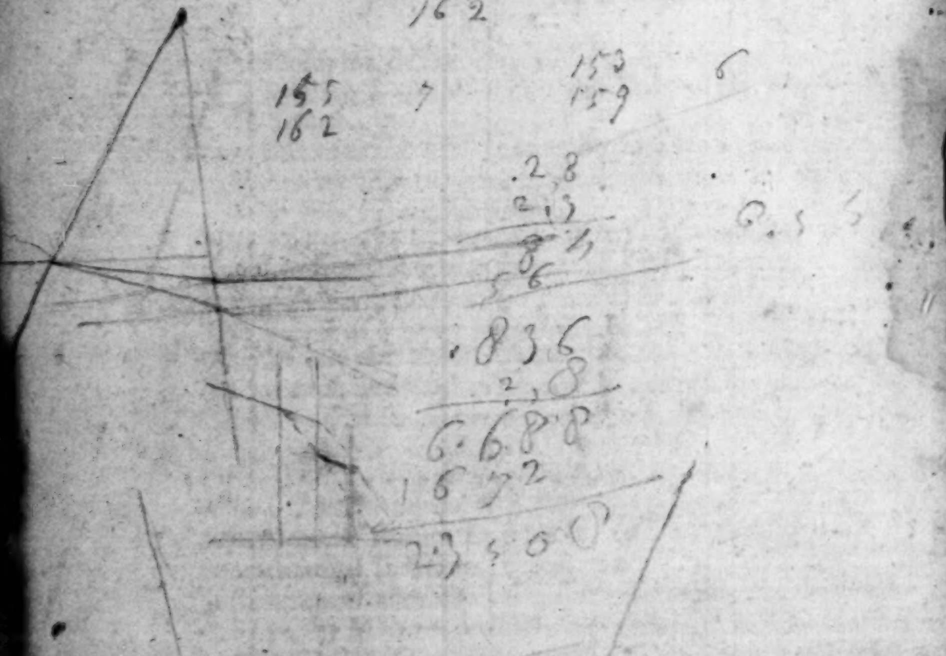

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ne whether if the Rays were made to pa

ro 2 Wedges (of diff. Glass & ∠s) A single Obj. G

woulde do

1.1×

1.1+1

.322

$$\frac{1.1 \times 1 =}{1.1+1 = 2.1 \times .6} \quad \frac{1.1}{1.26} = .88$$

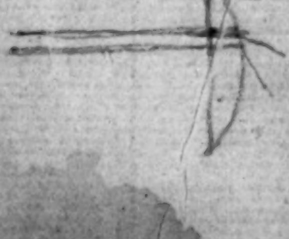
3 : 2 :: .88 : .587

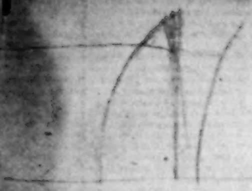
$$\begin{array}{r} .587 \\ .54 \\ \hline 2348 \\ 2935 \\ \hline R\ .31698 \end{array}$$

R2

1.1 × 1 × .3 = .33

Let the Crown glaſs be a plano, with its plain side outwards, for then the rays will not be diſpd till they are paſsed thro it; if so we shall not have so much divergency to correct.





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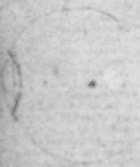
11.2 23.6

2.5 = 10



If it just beyond
the focus of ye Convex Objects will
be inverted

as Light is matter the harder

body is that it is to go through

perhaps the more Dim

E W

Error, For Brazil Pebble is hard yet very clear

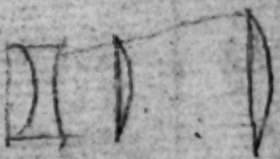
As 2 Convexes correct the

from the figure I think it

be best to have the Achrom

Obj. made thus

J. W. 24. Sep.



By putting the Convexes farther or nearer we

more exactly adjust the mag.d power to the concav

the focus of one = convex equal to the 2 C

Or perhaps it may be better to have 2 c